Contrary Negation as a combination of degree negation and a positive operator

Workshop: Diachronies of Negation — SALT 33

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The Square of Opposition and Adjectives

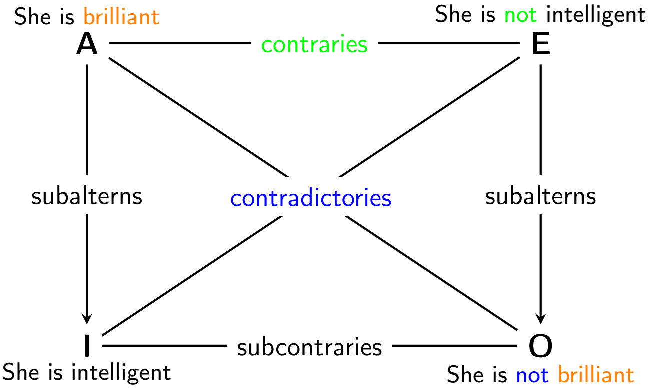
Gotzner et al. (2018):

"It might be tempting to take Aristotle's square of opposition as a template to be applied to all kinds of Horn scales. However, it is particularly important in the context of adjectival scales that the meaning relations of the square of opposition do not generalise. "

My Aim:

Show that they DO generalize.

The Problem



Gotzner et al. argue that **She is not intelligent** and **She is intelligent** are no contradiction in the system of meanings in combination with **brilliant**. How come?

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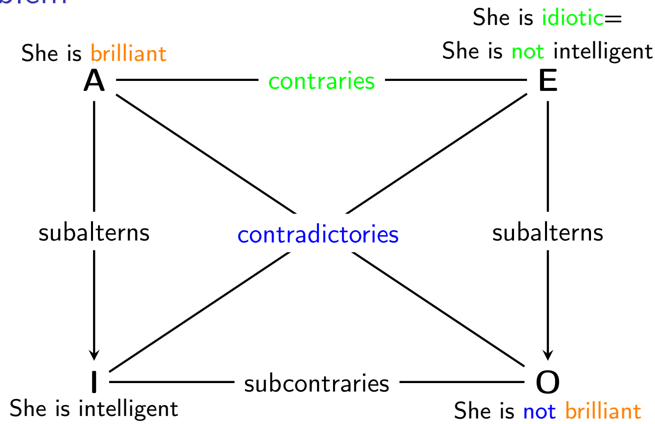

Gotzner et al. argue that **She is not intelligent** and **She is intelligent** are no contradiction in the system of meanings in combination with **brilliant**. How come?

The Square hammered flat

A measurement scale for brilliance

not intelligent not brilliant intelligent brilliant

- The contraries occupy the poles of the measurement scale.
- The sequence **not intelligent** has the same meaning as **idiotic**. But there is a tenison between the negation used implicitely in **idiotic** and the negation **not**. Two **nots**?
- The contraries restrict the regions where their negations apply.
- and therefore there is gap between between being idiotic and being intelligent.
- The measurement scale and the square of opposition somehow do not fit — at first sight.

Questions

- Is there a **contrary negation** besides classical negation? Are there differences in meaning between sentential negation and affixal negation?
- Are there two types of Square of Opposition, one for the interaction between negation and regular quantifiers and one for adjectives and their antonyms?
- How do Horn Scales and the related implicatures (or other inferences) fit into the picture?

Literature and Hypothesis

- **Literature**: There is no consensus in the literature. Binary/Non-binary negation, Weak/strong negation (Jacobs, 1991), classical/contrary negation (Horn, 1989; Horn and Wansing, 2020). Heim (2008),von Stechow (2009) posit a difference in scope in combination with a degree quantifier (see also DeClerq and Wyngared, 2017) .
- **Basic insight**: The problems for non-classical negation involve gradable predicates.
- **Analysis**: Seemingly non-binary negation can be decomposed into a (hidden) universal quantifier over degrees (the positive operator) and classical logical negation. The negation hypothesis remains valid. But negation is a type-flexible operator.

Map of the talk

- **Data**: Problematic cases are cases with gradable adjectives.
- **Theory**: Adopt a semantics for gradable adjectives, positive operator (universal quantifier) and a type-flexible negation operator.
- **Application**: differences in meaning are due to differnces in scope of classical negation with the positive operator: another scope effect.
- **Bigger picture 1**: Squares of opposition: Quantification and Predication with gradable adjectives are interrelated by negation.
- **Bigger picture 2**: Pragmatic inferences: Scalar implicatures, Negative Strengthening and Scale Reversal (Gotzner et al., 2018).

Affixal negation is resticted in scope

Scope effects: universal quantifier (Jacobs, 1991)

(1) **Alle Politiker sind nicht verheiratet.**
'Every politician is not married' ambiguous

(2) **Alle Politiker sind unverheiratet.**
'Every politician is unmarried' not ambiguous

Sentence (1) is ambiguous. Sentence (2) is not ambiguous. The scope of affixal negation is limited to the predicate **verheiratet** 'married'. Sentential negation interacts with quantifiers and allows for scope ambiguities.

Affixal negation may not cancel presuppositions

Scope effects: Presupposition triggers (Horn, 1989)

(3) **Der König von Frankreich ist nicht verheiratet. Es gibt gar keinen König von Frankreich.**
‘The king of France is not married. There is no king France.’

(4) **Der König von Frankreich ist unverheiratet.** #**Es gibt gar keinen König von Frankreich.**
'The king of France is unmarried. #There is no king of France.’

Sentence (3) allows for cancellation of the presupposition triggered by the definite description. Sentence (4) does not. The scope of affixal negation is limited to the predicate **verheiratet** ‘married’. Sentential negation interacts with definite descriptions and allows for scope ambiguities.

Solution: Negation is type-flexible

Analogous argument for conjunction in Partee and Rooth (1983)

- $[\![\textbf{nicht}_1]\!]^s = \lambda p.1 - p$ sentential negation
- $[\![\textbf{nicht}_2]\!]^s = \lambda P^{et}.\lambda x.[\![\textbf{nicht}_1]\!]^s(P(x))$ predicate negation

- $[\![\textbf{un-}]\!]^s = \lambda P^{et}.\lambda x.[\![\textbf{nicht}_1]\!]^s(P(x))$ affixal negation

- Semantic argument: narrow scope of **un-** wrt. overt quantifiers.
- Semantic/pragmatic argument: Presupposition cancellation is impossible with **un-**.
- Morphological argument: **un-** is a bound morpheme. Its application is limited to predicates.

Narrow scope negation: predicate negation = affixal negation

$$[\![\textbf{Alle Politiker sind unverheiratet}]\!]^{s^*}$$
$$= \vdash \text{All politicians are unmarried in } s^* \dashv$$

$$[\![\textbf{Alle Politiker}]\!]^{s^*}$$
$$= \lambda X. \vdash Pol_s \subseteq_{\downarrow} X \dashv$$

$$[\![\textbf{unverheiratet}]\!]^{s^*}$$
$$= \lambda x.[\![\textbf{nicht}_1]\!]^{s^*}(\lambda x. \vdash \text{x is married in } s^* \dashv (x))$$
$$= \lambda x.1 - \vdash \text{x is married in } s^* \dashv$$

$$[\![\textbf{un-}]\!]^{s^*}$$
$$= \lambda P^{et}.\lambda x.[\![\textbf{nicht}_1]\!]^{s^*}(P(x))$$

$$[\![\textbf{verheiratet}]\!]^{s^*}$$
$$= \lambda x. \vdash \text{x is married in } s^* \dashv$$

The Negation Hypothesis

Paraphrasing Jacobs 1991, 569

For every natural language *L* the following holds:
In an adequate semantic theory of *L*, every negative expression is represented (even if only partly) with classical logical negation.

Adequate semantic theory for German: method of direct interpretation as introduced in Zimmermann (2016).

Exceptions to the Neg Hypothesis

Allegedly no scope effect, still difference in meaning: Jacobs, 593

(5) **Der König von Frankreich erwies sich als ungebildet.**
'The king of France proved to be uneducated.' strong negation

(6) **Der König von Frankreich erwies sich als nicht gebildet.**
'The kind of France proved not to be educated.' weak negation

$\Rightarrow [\![\textbf{un-}_2]\!]^s \neq \lambda P^{et}.\lambda x.[\![\textbf{nicht}_1]\!]^s(P(x))$?

There is difference in meaning. The point: sentential negation does not interact scopally with the definite description. The **als**-phrase limits the scope of negation to the predicate **gebildet** 'educated', Jacobs (1991).

Interpretation of gradable adjectives

Following von Stechow (2009)

- The interpretation of gradable adjectives is associated with points on a scale (Cresswell, 1976).
- The points on the scale are called degrees and values of a measure function. i.e. numbers.
- **gebildet** 'educated' is interpreted with respect to a measure function: MEASURE$_{EDU}$. Some other adjectives may come with measure units: **long**, Length measured in meters: MEASURE$_{LENGHT,m}$.
- Adjectives express relations between an individual from a set of individuals A and a degree from the set of reels $\mathbb{R}$.

- $[\![\textbf{gebildet}]\!]^s = \lambda d.\lambda x.\vdash \text{MEASURE}_{\text{EDU}}(s)(x) \geq d \dashv$

The Positive, von Stechow (2009)

- The positive form of adjectives is complex, though. I adopt the view that there is an (invisible) positive morpheme that relates two sets of degrees. The second set is a contextually determined interval somewhere in the middle of the scale: delineation interval. That is: The positive morpheme corresponds to a universal quantifier.

Definition Positive

- $[\![\mathbf{POS}]\!]^{s,c} = \lambda D_1.\vdash NORM_s^c \subseteq\downarrow D_1\dashv$

- $[\![\textbf{DKF ist POS-gebildet}]\!]^{s,c} = 1$ iff $\forall d[d \in NORM_s^c \rightarrow \text{MEASURE}_{\text{EDU}}(s^*)(\text{TKF}) \geq d]$

Affixal Negation is type-flexible as well, von Stechow (2009)

- $[\![\textbf{gebildet}]\!]^s = \lambda d.\lambda x.\vdash \text{MEASURE}_{\text{EDU}}(s)(x) \geq d \dashv$
- $[\![\textbf{un-}_2]\!]^s = \lambda R^{d,et}.\lambda x.[\![\textbf{nicht}_1]\!]^s(R(d)(x))$ degree negation
- $[\![\textbf{un}_2\textbf{-gebildet}]\!]^s$
  $= \lambda d.\lambda x.1 - \vdash \text{MEASURE}_{\text{EDU}}(s)(x) \geq d \dashv$
  $= \lambda d.\lambda x.\vdash \text{MEASURE}_{\text{EDU}}(s)(x) < d \dashv$
- The Negation Hypothesis is still met.

Strong Negation: narrow scope wrt the positive operator

Negation negates the comparison relation: internal negation/degree negation

- $[\![$**DKF ist POS-gebildet**$]\!]^{s,c} = 1$ iff
  $\forall d[d \in NORM^c_s \rightarrow \mathrm{MEASURE}_{\mathrm{EDU}}(s^*)(\mathrm{TKF}) \geq d]$

- $[\![$**DKF ist POS-[[un-gebildet]]**$]\!]^{s,c} = 1$ iff
  $\forall d[d \in NORM^c_s \rightarrow NOT : \mathrm{MEASURE}_{\mathrm{EDU}}(s^*)(\mathrm{TKF}) \geq d]$
  iff $\forall d[d \in NORM^c_s \rightarrow \mathrm{MEASURE}_{\mathrm{EDU}}(s^*)(\mathrm{TKF}) < d]$
  relation flips

- Strong negation is a name for narrow scope negation with respect to a degree quantifier.

Strong Negation: narrow scope wrt the positive operator

Negation negates the comparison relation: internal negation/degree negation

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  relation flips

- Strong negation is a name for narrow scope negation with respect to a degree quantifier.

Weak negation: wide scope wrt the positive operator

Negation negates the positive operator (universal quantifier): external negation

- $[\![$**DKF ist POS-gebildet**$]\!]^{s,c} = 1$ iff
  $\forall d[d \in NORM_s^c \rightarrow \text{MEASURE}_{\text{EDU}}(s^*)(\text{TKF}) \geq d]$

- $[\![$**DKF ist nicht POS-gebildet**$]\!]^{s,c} = 1$ iff
  $\text{NOT} : \forall d[d \in NORM_s^c\text{: MEASURE}_{\text{EDU}}(s^*)(\text{TKF}) \geq d]$ iff
  $\exists d[d \in NORM_s^c\text{: MEASURE}_{\text{EDU}}(s^*)(\text{TKF}) < d]$
  quantifier and relation flips

- Weak negation is a name for wide scope negation with respect to a degree quantifier.

Weak negation: wide scope wrt the positive operator

Negation negates the positive operator (universal quantifier): external negation

- $[\![$**DKF ist POS-gebildet**$]\!]^{s,c} = 1$ iff
  $\forall d[d \in NORM_s^c \rightarrow \text{MEASURE}_{\text{EDU}}(s^*)(\text{TKF}) \geq d]$

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  $\exists d[d \in NORM_s^c\text{: MEASURE}_{\text{EDU}}(s^*)(\text{TKF}) < d]$
  quantifier and relation flips

- Weak negation is a name for wide scope negation with respect to a degree quantifier.

Dual Negation: external and internal negation combined

The positive operator (universal quantifier) intervenes between the two negations

- $[\![$**DKF ist POS-gebildet**$]\!]^{s,c} = 1$ iff for all degrees $d \in NORM_s^c \rightarrow \text{MEASURE}_{\text{EDU}}(s^*)(\text{TKF}) \geq d$
- $[\![$**DKF ist nicht [POS-[un-gebildet]]**$]\!]^{s,c} = 1$ iff NOT : d : $\forall d[d \in NORM_s^c \rightarrow \text{NOT} : \text{MEASURE}_{\text{EDU}}(s^*)(\text{TKF}) \geq d]$iff $\exists d[d \in NORM_s^c\text{: MEASURE}_{\text{EDU}}(s^*)(\text{TKF}) \geq d]$
  only quantifier flips
- Dual negation of a universal quantifier corresponds to existential quantification (An affirmative is expressed by the negation of the contrary: litotes)

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- $[\![$**DKF ist nicht [POS-[un-gebildet]]**$]\!]^{s,c} = 1$ iff NOT : d : $\forall d[d \in NORM^c_s \rightarrow \mathrm{NOT} : \mathrm{MEASURE}_{\mathrm{EDU}}(s^*)(\mathrm{TKF}) \geq d]$iff $\exists d[d \in NORM^c_s$: $\mathrm{MEASURE}_{\mathrm{EDU}}(s^*)(\mathrm{TKF}) \geq d]$
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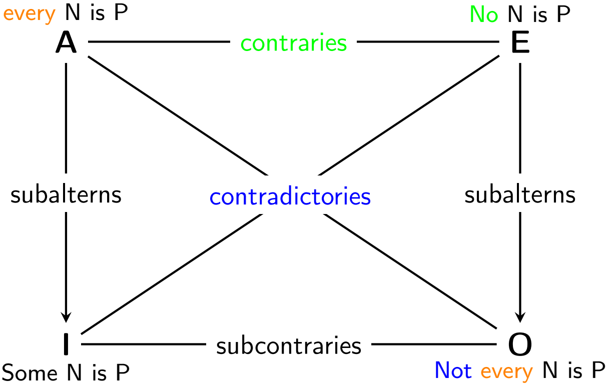
The Square of Opposition: Quantification in general

Aristotle, Horn (1989): Relations between Quantifications



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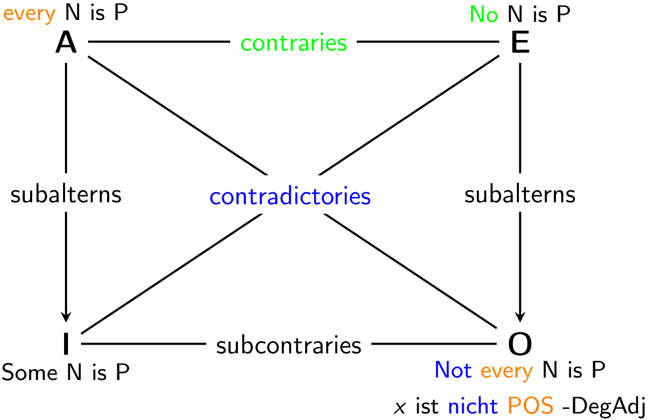

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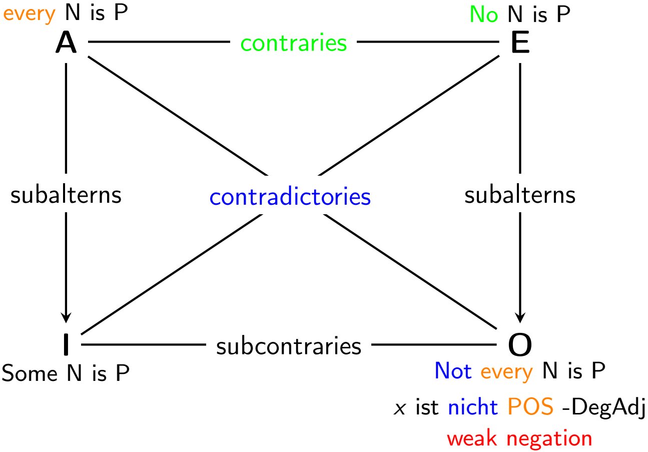

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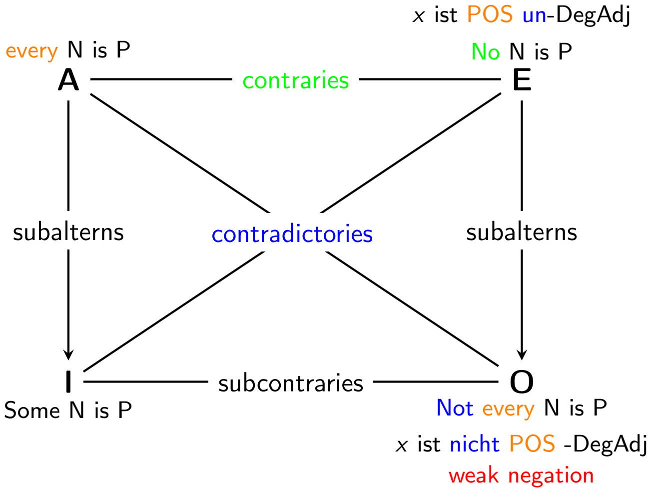

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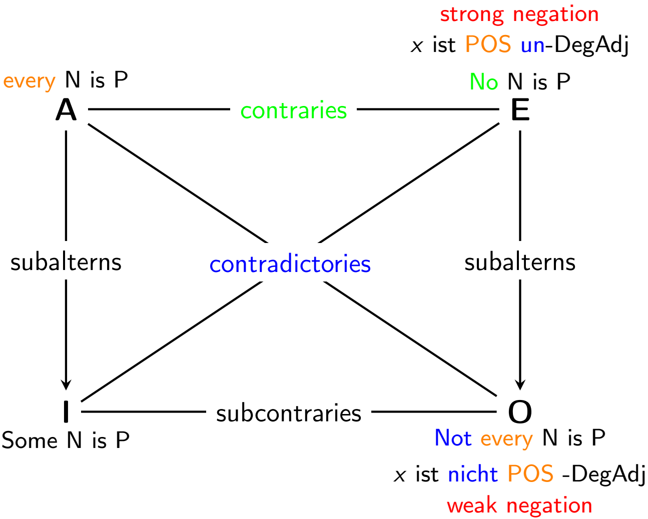

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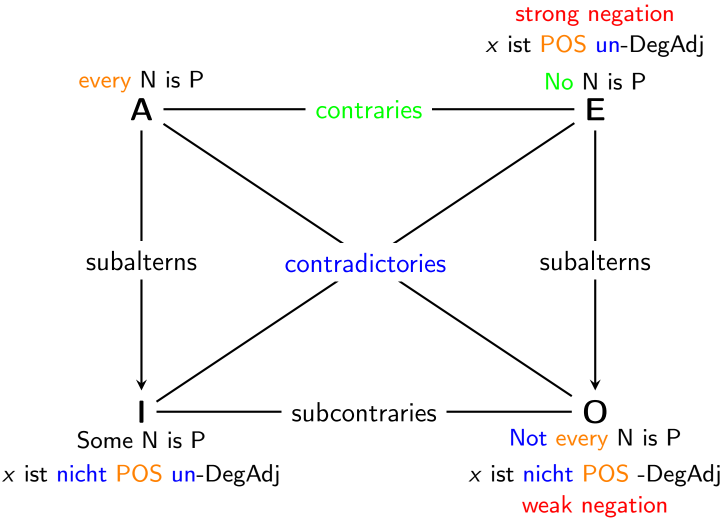

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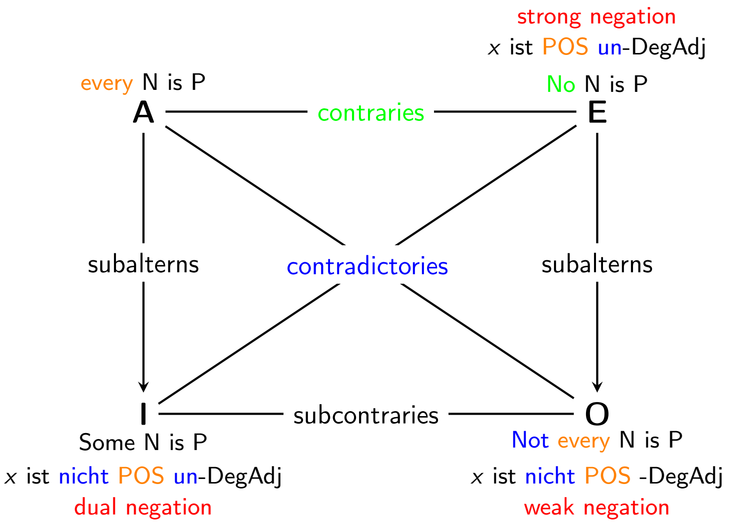
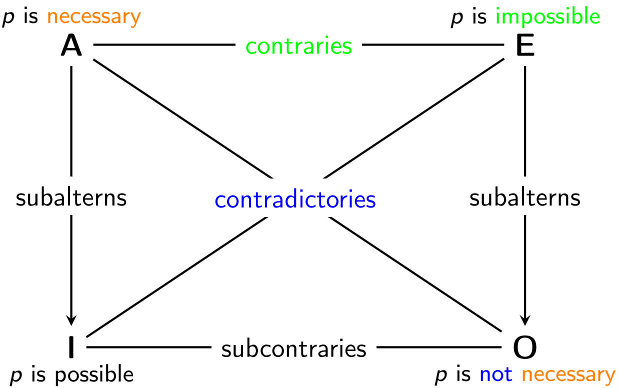


The Square of Opposition: Modals: 2 ways

Aristotle, Horn (1989): Relations between Quantifications



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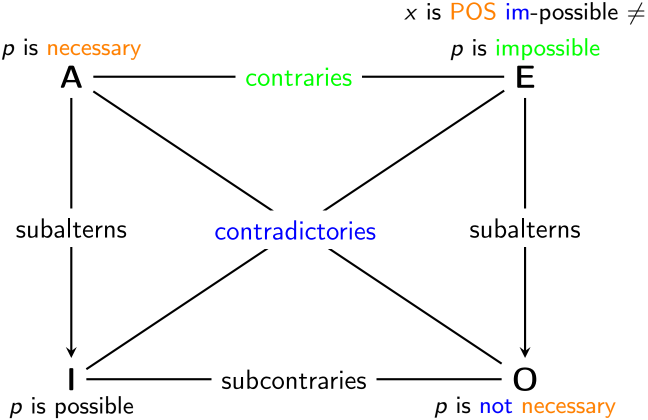

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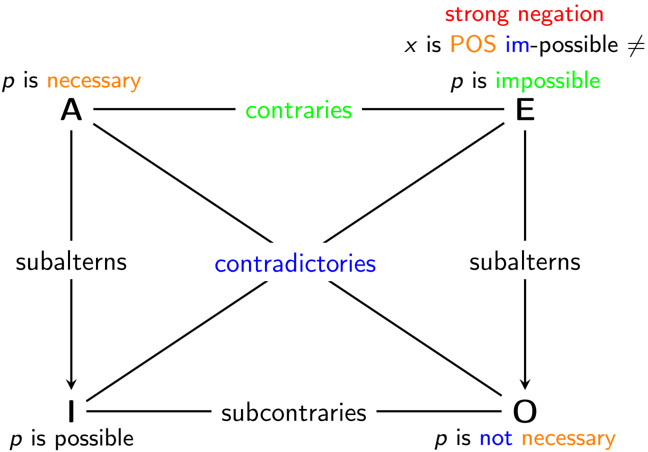

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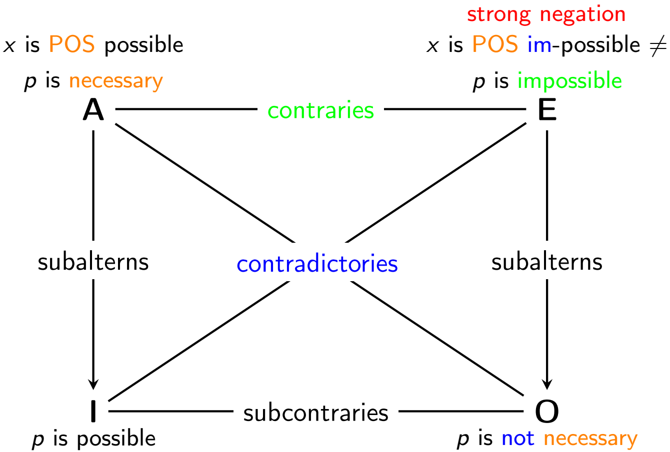

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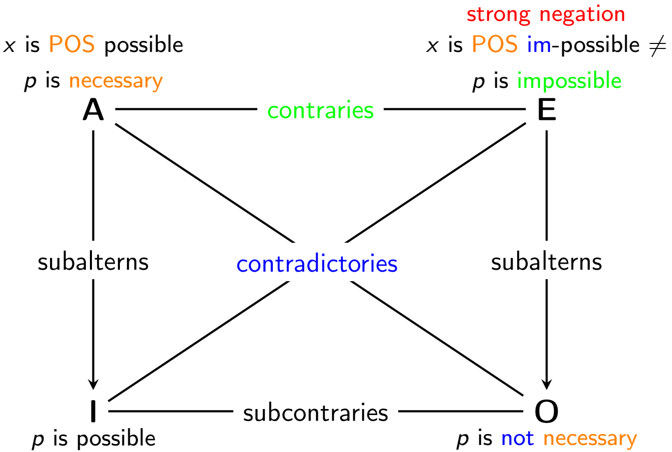

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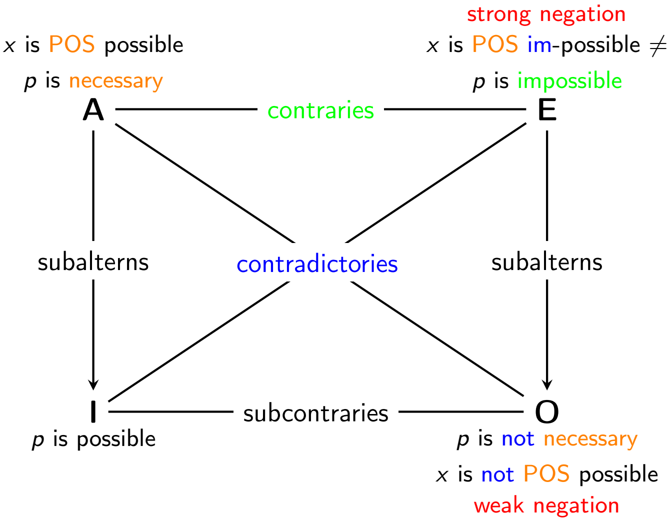

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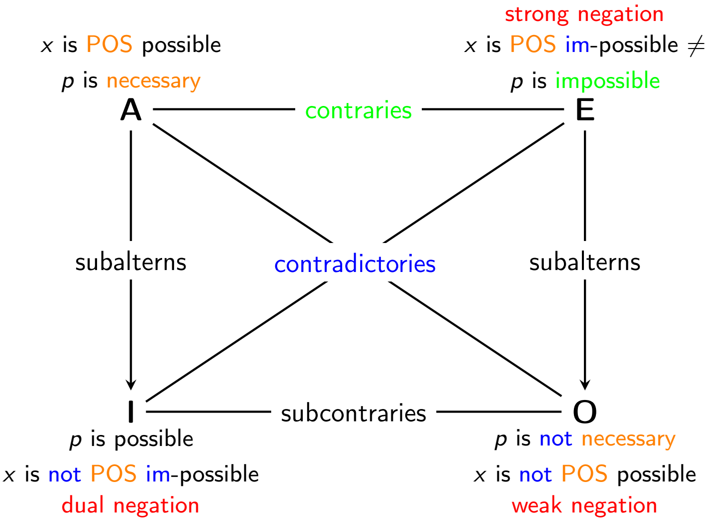
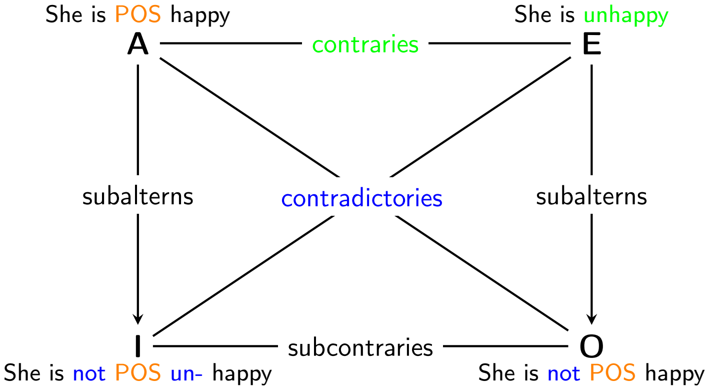


happy, unhappy and not happy



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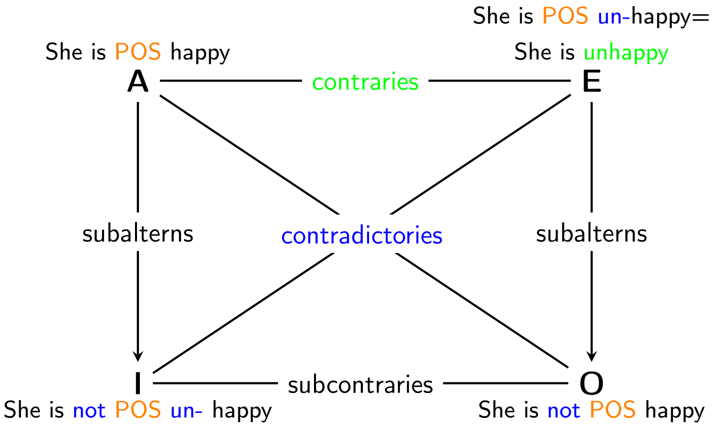
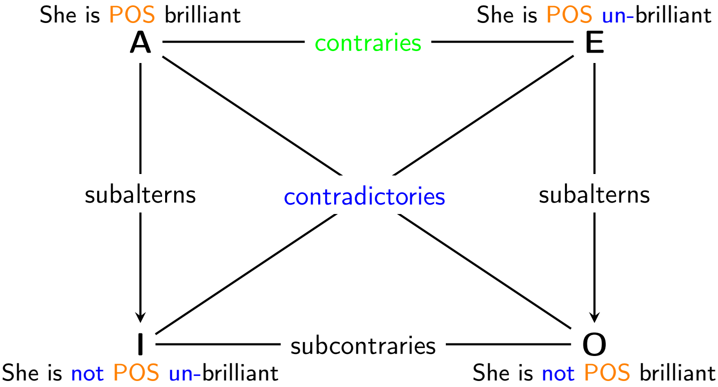


brilliant, **intelligent** and **idiotic**



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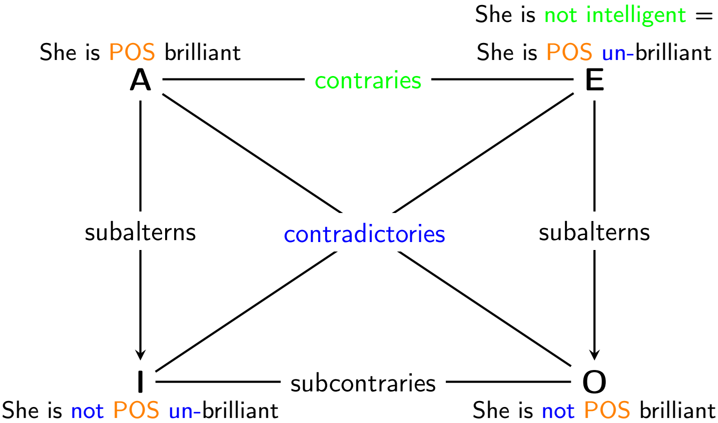

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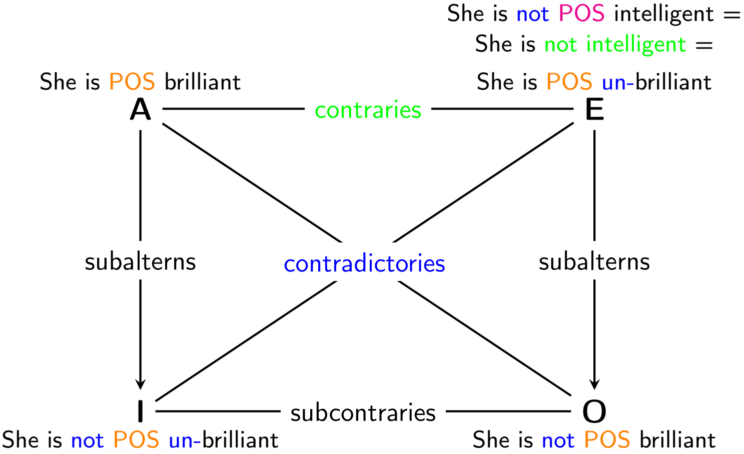
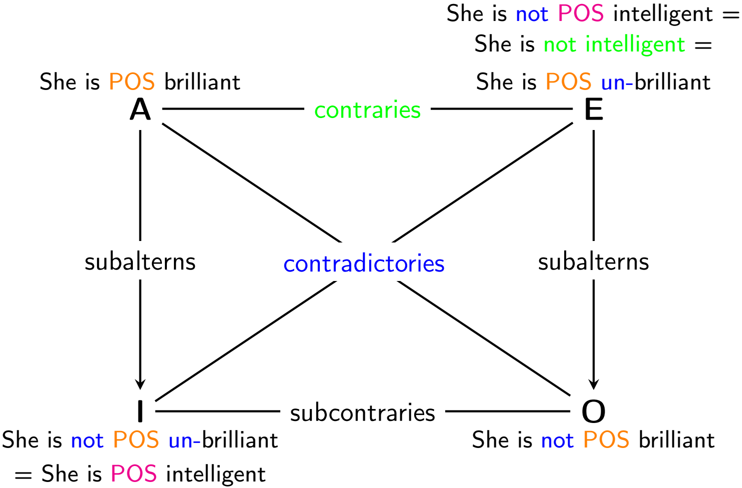


brilliant, intelligent and idiotic



Existential Degree Quantification

Definition Positive as an existential operator

- $[\![\mathbf{POS_2\text{-}}]\!]^{s,c} = \lambda D_1.\vdash NORM_s^c \cap \downarrow D_1 \neq \emptyset \dashv$

- $[\![\textbf{She is POS}_2\textbf{-intelligent}]\!]^{s,c} = 1$ iff
  $\exists d[d \in NORM_s^c \;\&\; \mathrm{MEASURE_{INTELL}}(s^*)(\mathrm{she}) \geq d]$

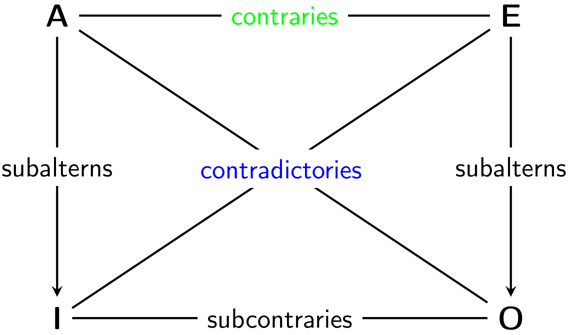
- **intelligent** patterns with **possible** in one reading and **some**.

Intermediate Summary

- Logical relations between (un)negated quantified statements may be visualized by the square of oppositions.
- One and the same adjective may participate in different entailment scales, though.
- There is no need for contrary negation: It is a scope issue and an issue of quantificational force.
- Proposal: The degree operator may shift from universal to existential meaning with different scope properties with respect to negation.
- The Square of Opposition does generalize to adjectival meanings.
- Two negations do not cancel out because a POS-Operator may intervene.

Bigger Picture: 3 Types of Inferences

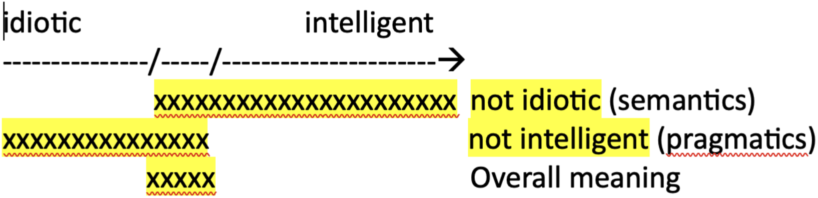
Gotzner et al. (2018) investigated pairs of adjectives with respect to inferences



- Scalar implicature, positive: I $\rightarrow$ O SI-pos
- Scalar implicature, negative: O $\rightarrow$ I SI-neg
- Negative strengthening: O $\rightarrow$ E NegS
- Experimental findings: SI-pos and SI-neg correlate, NegS and SI anti-correlate, for particular pairs of antonyms.

Scalar implicature, positive (SI-pos)

(7) **She is not idiotic** (I) $\hookrightarrow$ **She is not intelligent** (O)

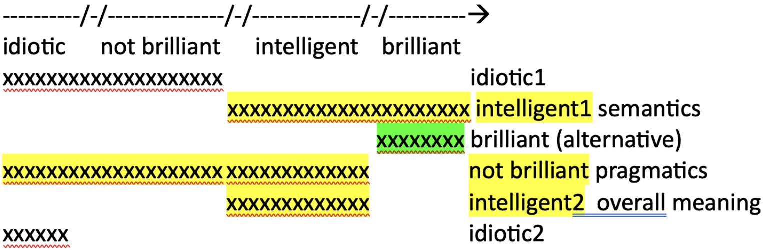


- Only 2 alternatives are matched with the measurement scale
- $\langle$**intelligent**$_A$, **not idiotic**$_I$, **idiotic**$_E$, **not intelligent**$_O\rangle$ (negation of the stronger proposition A)

Scalar implicature, positive (SI-pos)

medium size strengthening

(8) **She is intelligent (I) $\hookrightarrow$ She is not brilliant (O)**

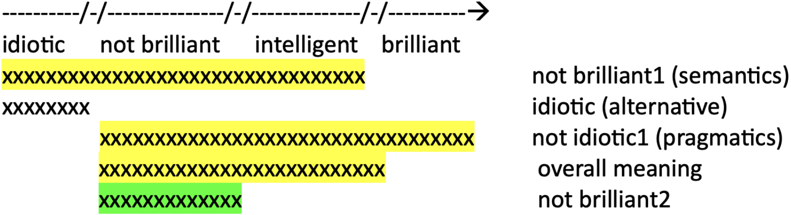


- 4 alternatives are matched with the measurement scale
- $\langle$**brilliant**$_A$, **intelligent**$_I$, **idiotic**$_E$, **not brilliant**$_O\rangle$

Scalar implicature, negative (SI-neg)

medium size strengthening

(9) **She is not brilliant** (O) $\hookrightarrow$ **She is not idiotic** (I)

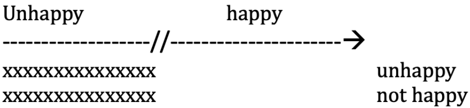


- 4 alternatives are matched with the measurement scale
- $\langle$**brilliant**$_A$, **not idiotic**$_I$, **not intelligent**$_E$, **not brilliant**$_O\rangle$

Negative Strengthening (NegS)

medium size elimination possible

(10) **He is not happy** (O) $\hookrightarrow$ **He is unhappy** (E)



- Horn's explanation: this is an instance of the inference pattern Modus Tollendo Ponens. It involves the law of excluded middle. the gap shrinks to a point: contradiction
- The Square of opposition collapses
- 2 alternatives are matched with the measurement scale
- $\langle$**happy**$_A$, **not unhappy**$_I$, **unhappy**$_E$, **not happy**$_O\rangle$

Negative Strengthening (NegS)

medium size elimination not possible

(11) **He is not happy** (O) $^*\hookrightarrow$ **He is unhappy** (E)

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unhappy   not happy   not unhappy   happy

xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx   not happy

xxxxxxxx   unhappy

- Law of excluded middle does not make sense in this picture.
- **Not unhappy** counts as an additional alternative
- 4 alternatives are matched with the measurement scale
- $\langle$**happy**$_A$, **not unhappy**$_I$, **unhappy**$_E$, **not happy**$_O\rangle$

Negative Strengthening (NegS)

medium size elimination not possible

(12) **He is not brilliant** (O) $* \hookrightarrow$ **He is idiotic** (E)

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idiotic not brilliant intelligent brilliant

xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx

xxxxxxxx

not brilliant result of strengthing idiotic

- Law of excluded middle does not make sense anymore in this picture.
- **intelligent** counts as an additional alternative
- 4 alternatives are matched with the measurement scale
- $\langle$**brilliant**$_A$, **intelligent**$_I$, **idiotic**$_E$, **not brilliant**$_O\rangle$

Conclusion

- The difference between sentential negation and affixal negation is a scope difference.
- There is only one negation: truth value reversal.
- Either a comparison relation is denied (introduced by the adjective) or the POS operator.
- It could be that **not** were type flexible all the way through: even a degree modifier. **un-** is never sentential (scopally inert) for type reasons. I proposed a POS operator with existential force and **not** is no degree modifier, by applying the Square of Opposition. The NegHypothesis remains valid.
- The account might be applied to the realm of scalar impicatures and makes the correct predictions.
- The availability of the implicatures and negative strengthening is a contextual effect of the restriction of the positive operator (the distance between two antonyms) and this matches the findings in Gotzner et al. (2018). Distance matters!

Thank you!

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