Negation and Antonymy*

Cécile Meier
Frankfurt University

Jan 2022

Abstract

Larry Horn discusses the longstanding question of how many types of negation there are from a semantic and pragmatic point of view in several places in his seminal work. And quite some researchers subscribe to the hypothesis that there is more to negation than its function in classical logic, i.e., truth value reversal or complementation. In this paper, I lay out a formal semantic background: a variety of truth-conditional semantics. I introduce classical negation as a type-flexible binary operator and the basics of degree semantics. Moreover, I discuss two problems for the formal semantic view that were used to show that negation is non-binary and no truth-functional operator. I show that the arguments for non-binary negation are not compelling, however. The problems concern constructions with gradable adjectives and a suitable analysis for gradable adjectives (containing quantifiers) explains the problems discussed nicely. Strong negation (contrary negation) and weak negation (contradictory negation) are shown to be names for a type of scope interaction of classical logical negation with the so-called positive operator (in the version of von Stechow) and double negation does not cancel out because the positive operator intervenes between the two occurrences of classical negation. The two types of negation are scope effects in the domain of degree semantics. And the complementation hypothesis (put forward by Joachim Jacobs) remains valid. This script was written for my students in the class "Antonyme" 2022 where we discussed several semantic approaches to antonyms.

*Thanks go to Caro Reinert for comments on an earlier version of this script and to my colleagues at Frankfurt University, especially Ede Zimmermann and Helmut Weiß, for discussion. The script, in fact, owes a lot to work by Arnim von Stechow and Irene Heim.

Contents

1 Introduction

The aim of this script is to investigate what is called contrary negation (see Horn & Wansing 2020: for an overview) or strong negation (Jacobs 1991) or non-binary negation. Contrary/strong/non-binary negation is sometimes used in contrast to classical logical negation and the basis for the relation between antonyms. And it plays a role in the interpretation of sentences containing gradable adjectives (Heim 2008, von Stechow 2009).

Sometimes, it is stated that formal semantics may not be able to capture all types of semantic oppositions (Storjohann 2015). This type of investigation entertains a wider notion of antonymy, however, than the one used here on the basis of contrariety and contradiction.[1] Antonymy is a relation between a pair of words. In a sentence, the elements of the pair may be substitued by each other and the propositions that the resulting two sentences express stand in the sense relation of contradiction or contrariety (Lyons 1977).

The structure of the script is as follows, I introduce the basic assumptions in formal semantics in Section 2. In Section 3, I add classical negation **not** to the framework. In Section 4, I discuss different types of oppositions. Contrariety is a type of opposition. Contrariety is defined as a sense relation between propositions and it is shown that only if the construction contains gradable predicates or nominal quantifiers, a contrariety is an intuitively good example for an opposition. This is an old insight, see Lyons (1977: Chapt.9) and Löbner (1999). Modal verbs, adverbs and other relevant expressions are missing from the discussion here. It has been argued that they are quantificational as well, and that they relate by negation. But the semantic machinery to interpret them is not the topic of this script. These expressions are intensional operators.[2] In addition, it is shown that the term 'contrariety' is sometimes used in cases that we also could call cases of presupposition failure. We discuss cases of presupposition failure and presupposition cancellation only superficially, though.

Sense relations like contradiction and contrariety are sometimes visualized by using the so-called Square of Opposition (Horn 1989). In Section 5, we discuss two types of squares: a quantificational one and one that seems not quantificational (at first sight) but where scales and points on a scale (i.e., degrees) play a role because gradable adjectives are involved in the sentences whoses senses (=the propositions expressed) are related. This section motivates looking more deeply into what it means to negate a predicate. In Section 6 we look at the different varieties of meaning of affixal negation **un-** and in turn at uses of the negation particle **not** as a predicate modifier. Negation is shown to be type-flexible and induces truth-value reversal

[1] Storjohann, for example, counts pairs like **dream** (n) vs. **anxiety** as antonyms. These examples are interesting on their own.

[2] Löbner (1999) is a good starting point for looking further into this topic.

or complementation dependent on the argument's type of meaning (set or function or truth value) and its semantic type: sentence or n-place predicate.

Affixal negation has a special use, though. It may be combined with gradable adjectives to induce degree set complementation. This is shown after introducing a formal semantics for gradable adjectives that uses measure functions motivated by a theory of measurement Krantz et al. (1971). The result of this compilation is that the examples that motivate contrary negation or strong negation (in contrast to classical negation) may be explained by a more fine-grained formal semantics that assumes that gradable adjectives are quantificational when used in the positive. The examples discussed in the literature are no counterexamples against the complementation hypothesis that states that negation is always truth functional. Negation is never non-binary but type-flexible. And this flexibility may have different effects. Seemingly non-binary readings of negation are in fact scope effects. This allows for a simple definition of antonymy: Two expressions are antonyms if their extensions are related by classical negation only.

2 Background

The purpose of formal semantics is to investigate the truth conditions of sentences and to predict how the truth conditions can be derived from the meanings of the parts of the sentence and how the parts are combined. That the meaning of sentences has something to do with truth conditions is based on the observation that two sentences may mean the same things if they are true under the same circumstances and false under the same circumstances, i.e. it is impossible that one element of a pair of synonymous (equivalent) sentences describes a given situation correctly and the other one does not (= Most Certain Principle, see Zimmermann & Sternefeld (2013), for example, as formulated by Cresswell 1982). Formal semantics deals only with those aspects of sentence meanings that may be caputred with truth conditions. In other words, formal semantics is a variety of *truth conditional semantics*.

The truth conditions of a sentence may just be written down in meta language in a certain pattern as in (1). Note the difference between object language (bold-faced) and meta language. The object language could be any language, but the meta language is the language we understand and use to explain things.

(1) The sentence **it is raining** is true if it is raining (and false otherwise).

Another notion for the truth-conditional aspects of sentence meaning is the notion of *content* (of a sentence). Truth conditions may be reconstructed set-theoretically. That two sentences have the same content, means that they apply to exactly the same *situations*, i.e. are true in the same situtations. We may collect those situations in which a sentence is true and identify the collection with the content. The content of a sentence is therefore equal to set of (possible) situations that this sentence describes correctly.[3] If we know what a sentence means, we have an ability (semantic competence): we are able to decide for any situation (or circumstance) whether that sentence is true or not in that situation. The content of the sentence **it is raining** may be written down as in (2) which may be abbreviated as in (3).

(2) The content of the sentence **it is raining** is equal to the set of situations in which it is true.

(3) $\|$**it is raining**$\| = \{s : \text{it is raining in } s\}$

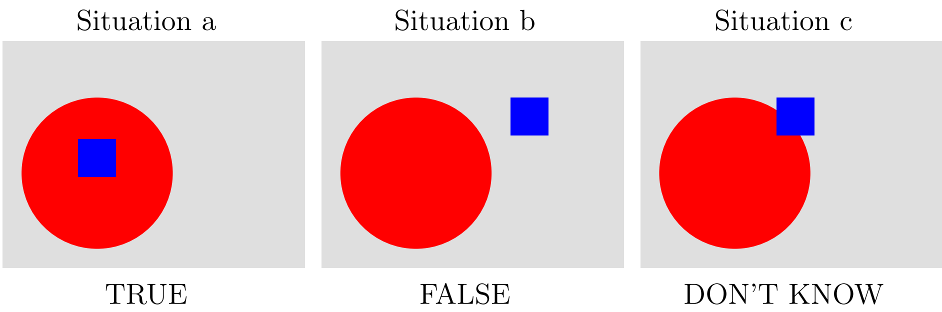
Sets of possible situations are called *propositions* (**D1.1**) and sentences express propositions (**D1.2**). The notion of proposition and content differ in that sentence content is connected to actual possible sentences used in normal utterance situations. Propositions don't have to represent sentences. A proposition is just a set of a certain type of elements: a set of situations. I follow Zimmermann (2021) in notation.

D1.1 A *proposition* is a set of situations – i.e., a set all of whose elements are situations.

D1.2 That a sentence S *expresses* a proposition p, means that p is the content of S.

Situations are defined as an *arbitrary* connected spatiotemporal region (Zimmermann 2021: p. 27). And the set of all situations is called *Logical Space* (abbreviated with *LS*). There are infinitely many possible situations in Logical Space. Two situations may just differ in a minimal detail. By way of illustration, consider the following sentence and three possible situations containing a red circle and a blue square.

(4) **The blue square is in the red circle.**



[3] Compare this view with the famous rule by Wittgenstein (1959): "Einen Satz verstehen, heißt wissen was der Fall ist, wenn er wahr ist." Translation: To understand the meaning of a sentence means to know what is the case if it is true.

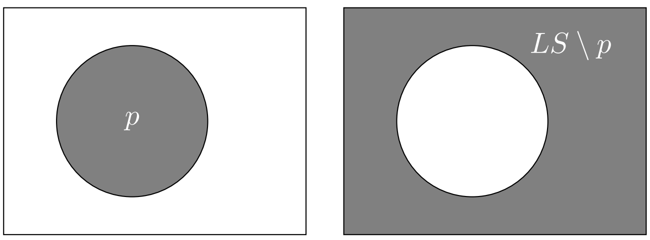
The pictures specify certain circumstances or abstract facts. Truth conditions describe circumstances. If the description is correct, the sentence is true. Everybody who understands the sentence in (4) is capable of deciding whether the sentence is true or not under these circumstances. If the truth conditions for the sentence are met, i. e., if the blue square is contained in the circle, we judge this sentence to be true. If however the truth conditions are not met and the blue square is definitely not within the circle area then the sentence is false. At this stage of the development of the theory we usually ignore borderline cases that are typical for the phenomenon of vagueness. Generally, it is assumed as a first idealization of the object of investigation that the Principle of Bivalence holds:

(5) *Principle of Bivalence* (also *Law of Non-Contradiction*)
Every declarative sentence is either true or false.

The content of a sentence divides the Logical Space in two areas: In the area that comprise the situations in which the sentence is true and the area that comprises the situations in which the sentence is false. The content partitions the Logical Space, in other words. And the borderlines between the areas are sharp.

This may be visualized by a one-set Venn Diagram. Venn diagrams visualize sets by a rectangle that contains (overlapping) circles. The interior of the circles contain the elements that are members of the set, and the exterior represents elements that are not members of the set. The rectangle represents the the Logical Space LS (all possible elements). A one-set Venn diagram has two regions. We may name the set of situations in such a diagram p (a proposition) and visualize this set p as well as its complement $LS \setminus p$ (the Logical Space without p) as a gray region in Logical Space as in the diagrams in (6).

(6) Set complementation



An equivalent way of expressing the meaning of a sentence is by means of a function that tells us for every situation in Logical Space whether this situation is described correctly or not, i.e., whether the sentence is true (abbreviated with the number 1) or false (abbreviated with the number 0). The numbers 0 and 1 are usually used in logic to represent truth and falsity. They are called truth values which goes back to Frege's work.

The bipartition of logical space induced by a proposition may be represented by a table as in (7). Let us assume that p contains those situtations in which it is raining and $LS \setminus p$ the situations in which it is not raining. This information may be listed in the table.

(7) $[\![$**it is raining**$]\!]$ =

Situation	Truth Value
s_0	1
s_1	1
s_2	0
s_3	0
s_4	1
s_5	1
s_6	0
s_7	0
...	

The table shows a function that assigns each situation in logical space a truth value, dependent on whether the sentence the function represents is true in that situation or not. The table states that at least the situations s_0, s_1, s_4 and s_5 belong to p if the table characterizes p and other situations s_2, s_3, s_6 and s_7 belong to the complement of p with respect to Logical Space. Representing a sentence meaning by a function (a set of pairs ⟨situation, truth value⟩ in the rows of the table) is set theoretically different form representing it as a set of situations. Functions as in (7) are called *sentence intensions*. The truth value that is assigned to a situation is called *extension*. The general definitions of intensions and extensions for sentences are in (**D1.6**) and (**D1.7**). The extension of a sentence in a situation is its truth value in that situation.

D1.6 The *intension* $[\![S]\!]$ of a (declarative) sentence S is the characteristic function of its content (relative to Logical Space), i.e., that function f with domain Logical Space and such that for every situation s the following holds:

$$f(s) = \begin{cases} 1 & \text{if } s \in \|S\| \\ 0 & \text{if } s \notin \|S\| \end{cases}$$

D1.7 The *extension* $[\![S]\!]^s$ of a (declarative) sentence S at a given situation s is the value its intension assigns to s, i.e.: $[\![S]\!]^s = [\![S]\!](s)$.

These definitions may be exemplified with our sentence **it is raining** as follows. (8) states the intension (dependent on the content that sentence expresses) and (9) its extension for a given situation s^*. It is obvious that intension and extension formalize the truth condtions for sentences.

(8) $[\![$**It is raining**$]\!]$ = the function f such that for every situation in

Logical Space the following holds:

$$f(s) = \begin{cases} 1 & \text{if } s \in \|\textbf{it is raining}\| \\ 0 & \text{if } s \notin \|\textbf{it is raining}\| \end{cases}$$

(9) $$[\![\textbf{it is raining}]\!]^{s^*} = \begin{cases} 1 & \text{if } s^* \in \|\textbf{it is raining}\| \\ 0 & \text{if } s^* \notin \|\textbf{it is raining}\| \end{cases}$$

The case distinction may be abbreviated as in (10) by introducting unconvential brackets $\vdash \ldots \dashv$. They serve to turn a truth condition into *a description of a truth value*. But since we do not know what the facts are in s^*, we cannot decide which truth value actually *is* the extension. But the extension of a sentence is always its truth value (in a concrete situation).

(10) $[\![\textbf{it is raining}]\!]^{s^*} = \vdash$it is raining in s$^*\dashv$

The definitions of content, intension and extension are interrelated. What the content of a sentence is, is derived from informal intuitive reasoning, namely that the meaning of a sentence may be captured by its truth conditions. Intensions are definied on the basis of the notion of content and tells us for any situation in Logical Space what its extension is, so that the extension of a sentence corresponds to the truth value that the sentence gets in a concrete situation. Content, intension and extension are means of capturing aspects of meaning. Formal semantics is a scientific tool to study meaning in detail. This paper builds upon Ede Zimmermann's introduction to formal semantics, i.e, direct interpretation (see also Zimmermann & Sternefeld 2013), and is written for readers that are interested in the treatment of negation in formal semantics. For the interpretation of quantifiers we depart from Zimmermann's account and turn to the introduction by Heim & Kratzer (1998) that include a introduction to quantifier interpretation and variable binding in direct interpretation. But the treatment is kept as simple as possible.

3 Negating sentences

In order to find out what sentence negation means we may apply the method of abstraction. The goal of the enterprise is to allow for compositional interpretation of negated sentences. The method of abstraction is based on the principle of compositionality. And we start with the meaning aspect called extension. The extension of a sentence is a truth value. And, the truth value of the negated sentence depends on the truth value of the corresponding unnegated sentence. As a starting point of our considertation we take the tree in (11). We know what the extension of a negated sentence is: a truth value. And we now what the extension of an unnegated sentence is: another truth value. But it is unknown what negation does.

(11)

$$[\![\textbf{It is not raining}]\!]^{s^*} = \vdash\text{it is not raining in } s^*\dashv$$

$$[\![\textbf{not}]\!]^{s^*} = \textbf{?}_1 \qquad [\![\textbf{it is raining}]\!]^{s^*} = \vdash\text{it is raining in } s^*\dashv$$

There is a systematic dependence of the truth value of the negated sentence from the truth value of the unnegated sentence. If the unnegated sentence is in fact true in the given situation s^* the negated one is false and vice versa. Abstracting away from the actual truth value a sentence may get in a given situation means here to consider the possiblilities that the sentence is true or false. It is obvious that negation reverses the truth value of the sentence it is applied to. Negation is a function f that gives for the truth value 1 the truth value 0 and for the truth value 0 the truth value 1.

We may capture this contribution by means of a table. Recall that all functions may be given in form of a table. The table eliminates our question mark $?_1$ in (11). This table is also called a truth table.

(12) $[\![\textbf{not}]\!]^s$ =

Truth Value	Truth Value
1	0
0	1

Another way to represent that function is the following. Negation is a sentence operator.

(13) $[\![\textbf{not}]\!]^s$ = the function f such that for any truth value t from the set of truth values T $(= \{1, 0\})$ the following holds:
$f(t) = \begin{cases} 1 & \text{if t = 0} \\ 0 & \text{if t = 1} \end{cases}$, for any situation $s \in LS$.

Abbreviated with a lambda term for functions we arrive at the following definition:

(14) $[\![\textbf{not}]\!]^s = \lambda t.1 - t$, for any situation $s \in LS$.

The tree in (11) may be completed as in (15). The truth conditions of the whole negated sentence are derived by applying the negation operation to the truth value that the sentence in a particular situation represents.

(15) $[\![\textbf{It is not raining}]\!]^{s^*}$
$= \vdash$it is not raining in $s^* \dashv$

$[\![\textbf{not}]\!]^{s^*}$ $= \lambda t.1 - t$

$[\![\textbf{it is raining}]\!]^{s^*}$ $= \vdash$it is raining in $s^* \dashv$

The intension of negation may be defined on the basis of this extension by abstracting away from given situations like s^* and considering any situation $s \in LS$. An intension is a function that assigns to every situation in the logical space its extension in that situation. Since the extension of negation does not vary from one situation to the next its intension is a constant function (typical for logical operators).

(16) $[\![\textbf{not}]\!]$ = the function f such that for any situation s from Logical Space the following holds: $f(s) = [\![\textbf{not}]\!]^s$

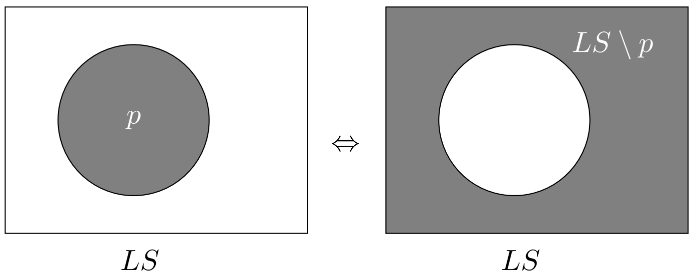
For any sentence represented by the intension captured by the table in (17) on the left there is an intension represented by the table in (17) on the right. And the two functions are related by truth-value reversal (i.e., negation).

(17)

Situation	Truth Value		Situation	Truth Value
s_0	1		s_0	0
s_1	1		s_1	0
s_2	0		s_2	1
s_3	0		s_3	1
s_4	1	$\Leftrightarrow$	s_4	0
s_5	1		s_5	0
s_6	0		s_6	1
s_7	0		s_7	1
...			...	

And since intensions correspond to propositions in logical space, negation relates a proposition to its complement in Logical Space. If the Venn diagram on the left visualizes the content of a sentence [any sentence that expresses the proposition p], the diagram on the right visualizes the content of its negation [the proposition $LS \setminus p$]. That is, the operation of negation is conceived as *set complementation*, as shown in Figure (18). At least the situations s_0, s_1, s_4 and s_5 are elements of the set named p and at least the situations s_2, s_3, s_6 and s_7 are elements of Logical Space but not of p, if that intension characterizes p.

(18)



From the point of view of sense relations, a sentence and its negation amounts to a *contradiction*, a type of opposition.

4 Contradiction and other types of oppositions

The literature distinguishes different types of oppositions ("Gegensätze"). And one question is how these oppositions relate to sentential negation expressed by **not** or whether they relate to sentential negation at all. The main distinction concerns *contradiction* and *contrariety* as *sense relations between propositions*. Sometimes *sub-contrariety* is added to the pair (Horn 1989).

Contradiction is defined as a relation between propositions as in (19). The propositions expressed by contradictory sentences are *disjoint*. And it is impossible that they are false simultaneously.

(19) The propositions expressed by two sentences S_1 and S_2 are *contradictory* if the following holds:

a. There is no situation $s \in LS$ such that $s \in \|S_1\|$ and $s \in \|S_2\|$: $\|S_1\| \cap \|S_2\| = \emptyset$, and

b. There is no situation $s \in LS$ such that $s \notin \|S_1\|$ and $s \notin \|S_2\|$: $\|S_1\| \cup \|S_2\| = LS$

If only the first clause (19a) of the definition (19) holds between two propositions, we call the relation *incompatibility* (Zimmermann 2021: p. 33). If two propositions are incompatible, the sentences that they express cannot be true simultaneously but it is not excluded that they are false simultaneously. Every contradiction is also an incompatibility. But not every incompatibility is a contradiction. The second case is sometimes called a contrariety.

(20) The propositions expressed by two sentences S_1 and S_2 are *contrary* if the following holds:

a. There is no situation $s \in LS$ such that $s \in \|S_1\|$ and $s \in \|S_2\|$: $\|S_1\| \cap \|S_2\| = \emptyset$, but

b. There is a situation $s \in LS$ such that $s \notin \|S_1\|$ and $s \notin \|S_2\|$: $\|S_1\| \cup \|S_2\| \neq LS$

If there is a situation in Logical Space that makes two sentences true they are called compatible. Again we may distinguish two cases, though. One is called sub-contrariety.

(21) The propositions expressed by two sentences S_1 and S_2 are *sub-contrary* if the following holds:

a. There is a situation $s \in LS$ such that $s \in \|S_1\|$ and $s \in \|S_2\|$: $\|S_1\| \cap \|S_2\| \neq \emptyset$, but

b. There is no situation $s \in LS$ such that $s \notin \|S_1\|$ and $s \notin \|S_2\|$: $\|S_1\| \cup \|S_2\| = LS$

The other case does not have a name besides compatibility. This is the relation usually holding between sentences in narrations and explanations.

(22) The propositions expressed by two sentences S_1 and S_2 are *compatible* if the following holds:

a. There is a situation $s \in LS$ such that $s \in \|S_1\|$ and $s \in \|S_2\|$: $\|S_1\| \cap \|S_2\| \neq \emptyset$, but

b. There is a situation $s \in LS$ such that $s \notin \|S_1\|$ and $s \notin \|S_2\|$: $\|S_1\| \cup \|S_2\| \neq LS$

All cases relate by a characterization of set intersection and set union relative to Logical Space. Only the frist three are related to oppositions in the literature.

We may visualize these sense relations expressing oppositions by means of two-set Venn diagrams, see Figure 1 and 2. Such a diagram has 4 regions corresponding to the possible operations on the two sets.[4] Let us name the left circle $\|S_1\|$ and the right circle $\|S_2\|$. One region corresponds to the intersection $\|S_1\| \cap \|S_2\|$, one region to the complement of the union of the two sets $LS \setminus \|S_1\| \cup \|S_2\|$, one to the difference $\|S_1\| \setminus \|S_2\|$ and one to the difference $\|S_2\| \setminus \|S_1\|$. The regions filled black do not contain any situations in the diagrams below. If two propositions are incompatible their intersection is empty and the complement of their union typically may be empty (contradition) or not (contrariety). If two propositions are compatible their intersection is non-empty and the complement of their union typically may be empty (sub-contrarity) or not.

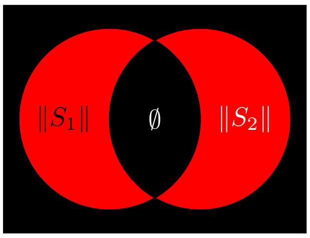
The sentences in (23) are good examples for sentences expressing an opposition: They are contradictions.

(23) a. **It is raining.** vs. **It is not raining.**

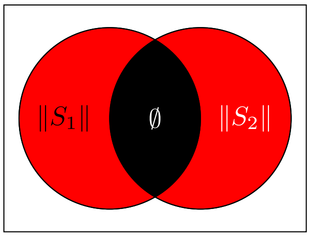
b. **Oddo is sad.** vs. **Oddo is not sad.**

c. **Mary is married.** vs. **Mary is unmarried.**

[4] Venn diagrams can be generalized to any number n of sets. The regions contained in an n-set diagram is a function of the number n of sets: 2^n. (If possible regions are missing in the illustration the diagram is called an *Euler diagram*.)

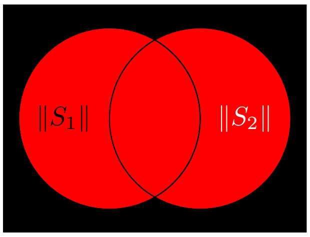


Contradiction



Contrariety

Figure 1: Incompatiblity



Subcontrariety

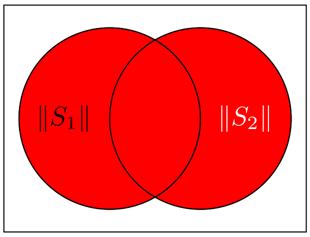


Figure 2: Compatibility

Sentences related to contrary propositions, however, are in many cases not really good examples for an opposition of any kind. Consider the sentences in (24) uttered at the same occasion. Replacing an expression by its co-hyponyme leads to contrariety but not necessarily to an opposition.

(24) a. **Oddo is a cat.** vs. **Oddo is a dog.**
b. **It is Wednesday today.** vs. **It is Monday today.**

Even more questionable are examples as in (25) where totally unrelated expressions are replaced by each other. I could point at a giraffe when uttering **this** and utter either (25a) or (25b). Both sentences are obviously false. But there are situations where I am right with either one of my utterances.

(25) a. **This is an elephant.**
b. **This is a square.**

Good examples for contrary oppositions come, in fact, from comparative and quantificational sentences Horn (1989). Consider first the sentences in (26) with two gradable adjectives that associate with the same scale. This type of sentence is called a predication (Zimmermann 2021: p. 52).

(26) a. **Oddo is sad.** vs. **Oddo is happy.**
b. **Oddo is big.** vs. **Oddo is small.**

The adjectives used in (26) may be used in comparisons as in **Oddo is**

smaller than Roy and analogous in all other examples. Those comparisons allow for the construction of a scale Krantz et al. (1971). With scales, we often associate points (or just numbers) that are related by an ordering relation among physical objects and the corresponding numbers. We may order individuals with respect to their height, comparing distances, or with respect to happyness or dirtyness. Often there is no measure unit as there is with heights or temperature, for example. The compared items do not have to be similar overall, but they share a property to a certain degree, named by the adjective and they are compared with respect to that property.

In fact, the sentences in (26) exemplify contraries: It is possible that both sentences are false. For (26a), this is the case in a situation where Oddo is indifferent: neither happy nor sad, just fine. For (26b), this would mean that Oddo is neither big nor small, just of average size. Adjectives like **happy** and **big** are called *relational adjectives.*

It is less clear whether a pair of sentences as in (27) with contrasting adjectives like **dirty** and **clean** (so-called *absolute adjectives*) also can both be false simultaneously. Rather there may be a zone of indifference between being clean and being dirty. And being clean is really the same thing as being not dirty.

(27) **Oddo is dirty.** vs. **Oddo is clean.**

The sentences might be an good example for the sense relation of contradiction. That there is a zone of indifference where we do not know which of the adjectives applies to an individual is a borderline case and could be related to the vagueness the adjective, see the discussion above on page 4 and on the Law of bivalence (5). And we could just ignore vagueness phenomena.

Weicker & Schulz (2020) argue that this kind of adjectives is not even vague. In fact, there is a larger discussion in the literature, whether the difference in contradiction and contrariety with gradable adjectives is just a difference in vagueness of the predicates involved in the sentences related. Krifka (2007) argues that a pragmatic reasoning leads to the impression that contraries are there in the first place. They are derived from contradictions.

Less controversial examples for contrariety involve *quantifications* as in (28). It is obviously possible that both sentences can be false. If only some of all the men are married but not all of them, none of the sentences is true.

(28) a. **Every man is married.**
b. **No man is married.**

(28) is an example that involves nominal quantifiers. But we observe the same phenomenon with sentential adverbs, modals and other quantificational intensional operators. Compare the examples in (29) and see Horn (1989) and Löbner (1999) for discussion. The two sentences all express contraries.

(29) a. **It is necessary that it is raining.** vs. **It is impossible that it is raining.**
b. **It has to rain.** vs. **It cannot rain.**
c. **The owner commanded the manager to advertize.** vs. **The owner prohibited the manager from advertizing.**

Two sentences are usually subcontraries of each other if they are contradictions of two contraries. In order to understand this it is worth considering the sentences in (30) and to compare them with the sentences in (28). (28ab) are contraries of each other. (30a) is contradictory to (28b) and (30b) is contradictory to (28a). (30a) is subcontrary to (30b).

(30) a. **Some men are married.**
b. **Not every man is married.**

Notice that sentence (30b) is equivalent to (31a). (31b) is equivalent to (30a).

(31) a. **Some men are unmarried.**
b. **Not every man is unmarried.**

This shows that universal (**every**/**all**) and existential quantifers (**a**/**some**) are interconnected by negation. The existential quantifier may be expressed by the universal quantifier and two instances of negation, a narrow scope negation and a wide scope negation, and vice versa. In formal terms, we may use the following notation in order to express the interrelatedness. It is even possible to capture the universal quantifier as a notational variant of the existential quantifier and two negations (Zimmermann 2021: p. 161). The translation of the English equivalents to the indefinite article and the unviersal quantificational determiner are the following. Universal and existential quantifier are called *dual operators*. They are the standard case of dual operators (Löbner 1999: p. 69).

(32) a. $|\textbf{some}_{indef}| = (\lambda Q.(\lambda P.\exists x[Q(x)\&P(x)]))$
b. $|\textbf{every}| = (\lambda Q.(\lambda P.\neg\exists x[Q(x)\&\neg P(x)]))$

In addition, the notion contrariety is used if a presupposition failure is detected in otherwise contradictory sentences. This use of contrariety goes back to Aristotle and it is usually exemplified with examples with proper names and definite descriptions. Consider (33).

(33) a. **The king of France is married.** vs. **The king of France is unmarried.**
b. **Socrates is ill.** vs. **Socrates is not ill.**

The proper name and the definite description presuppose that there is a unique referent in the situation in which the sentence is evaluated that has that name or that fits that description introduced in the noun phrase.

There are two semantic views on presupposition failure. In the first view, the sentence containing an unresolved presupposition trigger cannot be judged to be true or false in the first place. Such a sentence is undefined. This view goes back to Frege (1892) and was revived by Strawson (1950), but it contradicts the Law of Bivalence (5). This view is not compatible with our definition of contrariety. In the second view the non-existence of a referent for a proper name or a definite description for example may make a sentence just false. This view goes back to Russell (1905). This view is compatible with the definition of contrariety above, see also the treatment of definite descriptions in Zimmermann (2021). We will come back to presupposition phenomena when talking about differences between sentential negation and affixal negation and quickly tough on the pragmatic view of presupposition cancellation. But in principle, phenomena of presupposition are beyond the scope of this paper.

5 Two squares of opposition?

So far, we saw that quantifications and certain predications with or without negation are interrelated semantically by sense relations, i.e., different types of oppositions. Together, the propositions expressed may form a diagram: the square of opposition (see Parsons 2021, for the history, criticism, modifications and its application to reasoning). Horn uses the diagram in order to group expressions in quadruplets and explains gaps in lexicalization and a potential shift in meaning from a negated universal to a negated existential (Horn 1989). Horn distinguishes two separate types of quadruplets dependent on whether they are quantificational or not.

Let us consider quantification first: The quantificational determiners $\langle \textbf{every}_A, \textbf{some}_I, \textbf{no}_E, \textbf{not every}_O \rangle$ form a quadruplet and may relate two predicates S and P. The resulting examples may be placed in the corners of the diagram as in Figure 3.[5] The universal quantifier is placed in the upper left corner, called the A-corner, its negation in the lower right corner, called the O-corner. The existential quantifier is placed in the lower left corner, called the I-corner and its negation in the upper right corner, called the E-corner.[6] The A-O corners and the E-I corners are related by the sense relation contradiction, the A-E corners by contrariety and the I-O corners by sub-contrariety. Furthermore the A-I corners and the E-O corners are

[5]The template for the square of opposition in LaTeX is from the answer on a question on Stackexchange `https://tex.stackexchange.com/questions/594633/square-of-oppositions-diagram`

[6]The labels for the corners come form the Latin verbs **AffIrmo** ‘I affirm’ and **nEgO** ‘I deny’, see Horn (1989: 10ff.). They were introduced by commentators of Aristotles work. But note that the classification of the examples as “positive” and ”negative” is not always intuitive, if we use a formulation with the corresponding dual operators.

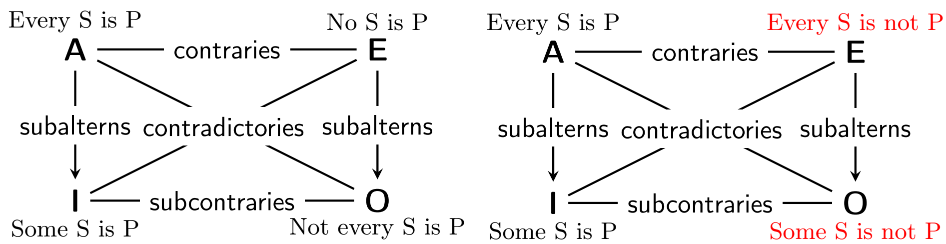


Figure 3: Square of opposition: Quantification

related by implication and sometimes called subalterns.

Since the universal quantifier and the existential quantifier are dual operators, the type of negation expressing a contrariety amounts to narrow scope of classical negation with respect to the quantifier and the type of negation expressing a contradiction amounts to wide scope negation with respect to the quantifier used. In the case of quantificational contraries, the difference between contrariety and contradiction is a scope effect of negation and the respective quantifer.

In the case of predications as in Figure 4, D refers to any individual and different types of predicates are related: The predicates $\langle$**good**$_A$, **not bad**$_I$, **bad**$_E$, **not good**$_O\rangle$ may serve as a good example for a quadruplet that illustrates the diagram. The different predicates have the common denominator P that may be replaced by the adjective **good**.[7] And **not-good** is lexicalized as **bad**. But what is the semantics of **not-**? The examples containing the corresponding pairs of antonyms and their negations are related by the same sense relations: The A-O corners and the E-I corners are related by the sense relation contradiction, the A-E corners by contrariety and the I-O corners by sub-contrariety. Furthermore the A-I corners and the E-O corners are related by implication and sometimes called subalterns. The main point is that there is a difference in meaning between **not good** and **bad** that may be expressed by two different types of negation. That is the difference is a difference in the semantics of the negation used in lexicalized negative predicates, not classical logical negation. Horn calls this type of negation *contrary negation*. If its semantics where classical negation the square would collapse since two classical negation cancel eachother out. I call this argument for contrary negation in contrast to classical negation *the argument from double negation*.

But, Horn refrains from giving a semantics for contrary negation in the non-quantificational cases. Horn & Wansing (2020) mention (and reject) a proposal for a *quasi-modal notion for contrary negation* akin to logical impossibility. So, the question how contrary negation may be captured semantically and how the adjectival cases relate to the quantifcational cases is

[7] Horn (1989: p. 8) mentions **black** and **white**, in addition.

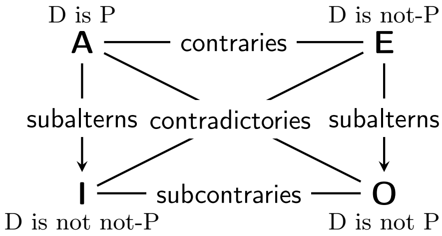


Figure 4: Square of opposition: Predication

an open question in Horn's work. Bierwisch (1989) argued that antonyms are related not by classical negation but *scale reversal*, not truth-value reversal. But in order to understand this we need some more background knowledge on scales and how they relate to language (see below).

Löbner (1999) also investigates quadruplets of expressions that are related by narrow scope or wide scope negation. He rejects the idea of a contrary negation. He considers an array of different types from the nominal, adverbial, verbal, and adjectival domain in addition to the well-known quantificational determiners. In his view, the members of the quadruplets are always related syntactically by *duality relations* between the respective existential and universal quantifier and classical negation. Pairing up dual expressions with their negative counterparts is a strategy for Löbner to detect quantificational expressions. And, it is interesting to see how Löbner's account relates to other proposals from degree semantics.

6 Negating predicates

Horn and Bierwisch argue for non-classical negation in formal semantics in addition to the classical one. Before we look at their arguments and proposals, we check out an account that uses *classical negation* in order to relate "opposite" pairs of expressions: (a) non-gradable predicates like the pair ⟨ **married**, **unmarried** ⟩ and then (b) gradable predicates like the pair ⟨ **happy**, **unhappy** ⟩. In a third step (c) we look at cases that are related by implicit negation as in the pair ⟨ **happy**, **sad** ⟩ for example.

6.1 Negated non-gradable predicates

So far we looked at sentences. Negation was described as an operation that turns a content into its opposite content. If applied to a sentence content, negation amounts to set complementation with respect to Logical Space. For sentence intension and sentence extension we observed that negation reverses the truth value. Set complementation (a set operation) and truth value reversal (a function) are interrelated by the respective aspects of sentential meanings: extensions vs. sentence contents. They do not differ semantically since the sentence content is characterized by its intension.

But what is the relation between parts of sentences? Consider the sentence in (34a). This sentence is equivalent to the sentence in (34b).

(34) a. **Mary is unmarried.**
b. **Mary is not married.**

Negation is either expressed by an affix **un-** or by the sentence particle **not**. Jacobs (1991) mentions two types of potential differences between sentential negation and affixal negation. These are differences that we may align with *scopal differences* of the two negations. Consider the sentences in (35). Whereas (35a) is ambiguous, (35b) is not. Affixal negation seems not to interact with other quantifiers in the sentence. (35a) means either "For every politician it is not the case that he/she is married" (narrow scope reading for negation with respect to the quantifier) or it means "It is not the case that every politician is married" (wide scope reading for negation).[8] (35b) only has the narrow scope reading for negation with respect to the quantifier. Sentential negation allows for scope ambiguities in relation with quantifiers, affixal negation does not.

(35) a. **Every politician is not married.**
b. **Every politician is unmarried.**

The second type of difference relates to differences in interaction with presupposition triggers. Consider the sentences in (36).

(36) a. **The king of France is not married.**
b. **The king of France is unmarried.**

The main point of these examples is that there is no referent for the definite desciption **the king of France** in the actual utterance situation.[9] France is no monarchy. Both sentences seem uninterpretable (or false) at first sight. The observation is, however, that (36a) may be continued with a sentence like (37). This observation is due to Russell (1905). And maybe there is some intonational effort necessary in order to follow Russell in his observation. Therefore, there must be a (maybe far-fetched) interpretation that makes (36a) true if it is possible to continue this sentence meaningfully. It is obvious that it is not possible to continue sentence (36b) with affixal negation with the sentence in (37). There is no interpretation for this sentence that makes (36b) true if there is no king of France.

[8] See Büring (2003) for how the two readings relate to different intonation patterns and why these have a disambiguating effect.

[9] A description that has no obvious referent in the situation in which it is evaluated is called *empty description*.

(37) **There is no king of France.**

There are several solutions on the market for this puzzle. One possibility is to assume that the definite description is interpreted quantificationally as argued for in Zimmermann (2021) and already mentioned above. If we do so, then the definite description is just an existential quantifier (with a uniqueness condition) and this quantifier may interact with sentence negation but not with affixal negation. This explanation is the one Russell preferred.

Presupposition theories, on the other hand, assume that definite descriptions are presupposition triggers. And the information contained in the presupposition has a different status than the information that is part of the sentence. Whereas sentence negation may target both types of information, affixal negation may only target the information contributed by the word that. A definite description may trigger an existence and a uniqueness presupposition, i.e., a pre-condition, for example, on the knowlegde the dialog partners have (Heim 1986/87). The difference between (36a/b) then is that in (36a), the existence and uniqueness condition maybe denied by sentence negation. And this denial reading may be confirmed by a sentence like (37). In (36b), however, the existence and uniqueness condition *must* hold if the sentence is interpretable in a dialogue situation. Information structure may influence whether sentence negation is able to target a presupposition (Hajičová 1994, Meier & Kohlhoff 1997, Beaver & Zeevat 2007).

We won't bother with a decision on this specific matter and assume that definite descriptions may be interpretated quantificationally as assumed in Zimmermann (2021). In this view, the difference in (36) is not different form the first example above with a universal quantifier and may be attributed to a scopal ambiguity that is possible with sentence negation but not with affixal negation.

In order to determine the semantic contribution of **un-** we may use the method of abstraction again. We derive its extension. Consider the tree diagram in (38).

(38)

$[\![\textbf{Mary is unmarried}]\!]^{s^*}$
$= \vdash$Mary is unmarried in $s^* \dashv$
$= 1 - \vdash$Mary is married in $s^* \dashv$

- $[\![\textbf{Mary}]\!]^{s^*}$
  $=$ Mary
- $[\![\textbf{unmarried}]\!]^{s^*}$
  $= \lambda x.1 - \vdash x$ is married in $s^* \dashv$
  - $[\![\textbf{un-}]\!]^{s^*}$
    $=$ **?**
  - $[\![\textbf{married}]\!]^{s^*}$
    $= \lambda x. \vdash x$ is married in $s^* \dashv$

In order to eliminate the question mark, it is impossible to interpret the affixal negation all the same as sentential negation. The exstension of sentential negation is an operator that combines with a truth value to give another truth value. But the argument of $[\![\textbf{un-}]\!]^s$ is a one-place predicate (a function), not a truth value. The argument of the affixal negation (if interpreted just like sentential negation) would be of the wrong semantic type and their combination produces a type mismatch.[10] It is however possible to derive the extension of affixal negation from the extension of sentence negation. If the extension of **Mary is unmarried** and **Mary is not married** is the same, we may translate $[\![\textbf{Mary is unmarried}]\!]^{s^*}$ with 1-⊢Mary is married in s^*⊣. The predicate extension is determined by abstraction from the meaning of the subject of the sentence and is represented by a the function represented in (39a). This representation may be reformulated on the basis of the extension of the predicate.

(39) $[\![\textbf{unmarried}]\!]^{s^*}$

a. $= \lambda x.1 - \vdash x$ is married in $s^* \dashv$

b. $= \lambda x.1 - [\![\textbf{married}]\!]^{s^*}(x)$

From the last line in (39) we may derive the contribution of affixal negation **un-** by functional abstraction. It assigns predicate extensions to predicate extensions. It basically reverses the truth value that any predicate would assign to an individual if the predicate were applied to that individual. Compare the extension in (40).

(40) $[\![\textbf{un-}]\!]^s = \lambda P^{et}.\lambda x.1 - P(x)$

This predicate modifier may be restated using the extension of sentence negation. The extension of sentence negation was defined as simple truth value reversal. Affixal negation turns out to be a certain use of sentential negation and not different from ordinary logical negation in principle.

(41) $[\![\textbf{un-}]\!]^s = \lambda P^{et}.\lambda x.[\![\textbf{not}]\!]^s(P(x))$

So, we are ready to eliminate the question mark in the tree diagram in (38). Changing the type of **not** from a sentential operator to a predicate operator avoids the type mismatch observed above. Moreover, it explains why **un-** does not participate in scopal ambiguities with other quantifers and definite descriptions. **un-** is required to combine with a one-place predicate. This forces any quantificational expression in subject position to have wide scope with respect to negation.[11] In addition, we may define a predicate modifier

[10] Note that types may be conceived as semantic labels for sets of expressions that share the functional operations that may be performed with it, see Zimmermann (2021) for details on type theory in connection with type logic and indirect interpretation.

[11] The arguments are related and analogous to an argument for type-flexible conjunction. Partee & Rooth (1983) argued that a sentence like **Mary caught and ate a fish** may not be derived by conjunction reduction. It must be possible for the indefinite **a fish** to

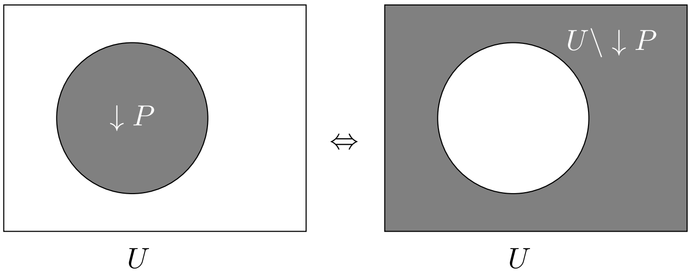


Figure 5: Negation relates the sets characterized by two predicates

not$_P$, a synonym of affixal negation **un-**, and higher type homonyms if putatively sentential negation combines with more complex predicates like **kiss** or **give**. Type flexibility for negation may capture the insight from syntax, that negation particle likes to occur close to the verbal (or adjectival predicate in a sentence.

(42) $[\![\mathbf{not}_{P1}]\!]^s = \lambda P^{et}.\lambda x.[\![\mathbf{not}]\!]^s(P(x))$
$[\![\mathbf{not}_{P2}]\!]^s = \lambda P^{e(et)}.\lambda y.\lambda x.[\![\mathbf{not}]\!]^s(P(y)(x))$
$[\![\mathbf{not}_{P3}]\!]^s = \lambda P^{e(e(et))}.\lambda z.\lambda y.\lambda x.[\![\mathbf{not}]\!]^s(P(z)(y)(x))$

Since all these definitions build upon the classical logical operation of truth-value reversal, the effect of two negations cancels out (Law of double negation).

(43) $[\![\mathbf{not}]\!]^s([\![\mathbf{not}]\!]^s(p)) = p$, for any truth value p.

This explains the equivalence between the following sentences, for example. Note that the square of opposition collapses because of the equivalences in (44).

(44) a. **Mary is not unmarried.**
b. **Mary is not not married.**
c. **Mary is married.**

The set P that a particular predicate extension like $[\![\mathbf{married}]\!]^{s^*}$ characterizes may be visualized as a set of individuals in a one-set Venn diagram as in Figure 5. It partitions the set of all individuals U (the universe of discourse in s^*) into two areas: one area that covers the married individuals and the rest. A predicate negated by **un-** or **not**$_{P1}$ characterizes the complement set in U of the the set that the unnegated predicate characterizes.[12]

Sorting out the individuals with respect to whether they are married or

have wide scope with respect to conjuntion.

[12] The downarrow is defined as an operation that allows for assigning the set characterized by f, if f is a characterizing function: $\downarrow f = \{x \mid f(x) = 1\}$, see Zimmermann (2021: p. 59 D2.3).

unmarried corresponds to what is called a nominal scale that is also used in data measurement in behavioral sciences. And negation is one means to come up with a bipartition. The sets arranged in nominal scales have no quantitative significance by their own.

Negation may be seen as triggering complementation even if it is a predicate modifier and not a sentential operator. Complementation on the level of the sets characterized by a predicate corresponds to contradiction on the level of the respective proposition.

Jacobs (1991: p. 569) states the following hypothesis: We may call this hypothesis the *Complementation Hypothesis* (HNEG). NEG represents the extension of sentence negation $[\![\mathbf{not}]\!]^s$ in English (and its homonyms and synonyms) and its semantics corresponds to classical logical negation (truth value reversal/complementation). But negation may be type-flexible.

(45) For every natural language L holds that in an adequate semantic theory, every occurrence of negation is representable with NEG.

The main question is whether HNEG is true or whether there are counterexamples that are not possible to be explained away.

It is unclear whether **un-** also applies to other predicates than just one-place predicates. In principle we expect that **un-** is applicable to two-place predicates, as well. In the absence of an English example, we may turn to German. **stolz auf** 'proud of', for example, might be a case in question where **un-** combines with a two-place predicate. Interestingly this type of construction is only acceptable if negation is doubled, as in (46).

(46) **Maria ist nicht unstolz auf ihren Sohn.**
Mary is not un-proud of her son
'Mary isn't not proud of her son.'

Note that the construction might be an instance of the rhetorical figure litotes (Horn 2017). This means that the sentence has a reading where (46) is not equivalent to the unnegated construction in (47). Negation does not cancel out. This observation may give rise to doubts about the correctness of HNEG. If negation does not cancel out, one of its interpretations might be not logical. We already mentioned this problem and called it *the argument from double negation.*

(47) **Maria ist stolz auf ihren Sohn.**
Mary is proud of her son
'Mary is proud of her son.'

The second argument comes from the interaction of negation with quantifiers. Jacobs discusses an example from German as in (48). For both sentence it is not possible to understand what is meant by them if there is no king of France. This shows that negation has in both sentences narrow scope.

Sentence negation is so-to-speak trapped in the **als**-phrase. But if there were a king of France, there is still a difference in meaning in this minimal pair. Jacobs finds that the negation that the sentential particle expresses is *weaker* than the negation affixal negation expresses. The affixal negation is *stronger*. We dubb this argument *the argument from scope*.

(48) a. **Der König von Frankreich erwies sich als**
the king of France turned_out himself as
ungebildet.
uneducated
'The king of France tourned out to be uneducated.'

b. **Der König von Frankreich erwies sich als**
the king of France turned_out himself as
nicht gebildet.
not educated
'The king of France tourned out to be not educated.'

Our question is whether this distinction is the same distinction that was introduced in the last section. Do differences in strength correlate to our difference between contradiction and contrariety in connection with gradable predicates? Contrary negation is stronger than contradictory negation in the sense that whenever a contrary negation is true the contradictory negation is true as well. There is an implication relation between the I- and the O-corner in the Logical square.

In the next step, we look more closely at the interpretation of gradable predicates. We will see that there are different classes of such predicates. The big question is whether we may explain the phenomena observed by Jacobs and Horn as a semantic phenomena.

6.2 Negated gradable predicates

Before looking at negated gradable predicates, we introduce the interpretation for their unnegated versions. There are several accounts on the market to explain what the semantic contribution of gradable adjectives are and we will concentrate on those. Sapir (1944) is often cited to be the first to mention that adjectives (in English) that are gradable express comparisons even in the positive. **Happy** is a gradable adjective as witnessed by the sentences in (49). (49a) may mean that Amy is happier than some standard of comparison.

(49) a. **Amy is happy.**
b. **Amy is happier than Dorit.**
c. **Amy is the happiest student.**
d. **Amy is very happy.**

Gradable adjectives may be used in the positive, the comparative and in the superlative. This morphological criterion is usually used to identify adjectives as a class. A syntactic criterion for deciding that an expression is an adjective is that it may occur in constructions of nominal modification as in **happy student**. In addition they can be used with intensifiers like **very**. **married** is not gradable as witnessed by the examples in (50). It is nevertheless an adjective as witnessed by nominal modification, e.g. **married student**. (50b-d) are a little odd unless they are reinterpreted as gradable adjectives. If reinterpreted, they signal what (50a) signals and in addition that Amy is (very) close(er) to her partner (or anybody else) than what is normal. The hearer may ask herself why the speaker is breaking the grammatical rules and explains this by assuming that the adjective is used with additional meaning. The adjective then shifts in meaning. The term is *coercion* if this type of shift is established as a semantic process.

(50) a. **Amy is married.**
b. #**Amy is more married than Dorit.**
c. #**Amy is the most married student.**
d. #**Amy is very married.**

Cresswell (1976) noted that if we express a comparison, "we have points on a scale in mind". So it is plausible to take this insight as a starting point of our considertations for the interpretation of adjectives. We all are familiar with scales in everyday life. Scales play a role in measurement: We measure height and temperature, and also happiness (but usually not "marriedness"). What are scales from a theoretical point of view?

6.3 Scales and measure functions

In order to capture what scales are and how they relate to adjectival meanings, we start with a set of individuals where the individuals are ordered according to some attribute or quantifiable property. For instance, it is possible to bring people into an order according to how happy they are or locations according to how warm it is there or we may order color pencils according to their length, etc.. In each example, we refer to an order on a set of individuals. Language is not necessary in order to establish the order, but it may help to express ourselves.

Formally, an *order is a two-place relation* R on a set of individuals A (Partee, ter Meulen & Wall 1990: pp. 47, 207). The relation has special properties. Consider the following example: R_1 is the relation 'is less happy than or equal' in the set A_1, where $A_1 = \{$Amy, Bernice, Celine, Dorit, Elba$\}$ such that Amy is happier than Bernice, Bernice is happier than Celine, Celine is happier than Dorit, and Dorit is happier than Elba. And everybody is a happy as she is. Such a relation is an empirical order. It depends on the facts how happy each individual from the set A_1 is. This type of relation is a weak order. We could extend A_1 to the set of all people.

The relation R_1 may be diagrammed as in Figure 6, with **a** representing 'Amy', **b** 'Bernice', **c** 'Celine', **d** 'Dorit' and **e** 'Elba'. The relation is antisymmetric (represented by the straight arrows) and reflexive (represented by the loop) and transitive (represented by the longer bent arrows). Elba is the least happy individual and Amy the happiest. If a pair of individuals $\langle x, y\rangle$ is element of R, we say that the first coordinate x of the pair precedes the second y. Instead of the general symbol R for relations, the symbol $\preceq$ is often used that signals precedence with respect to some attribute like happiness.

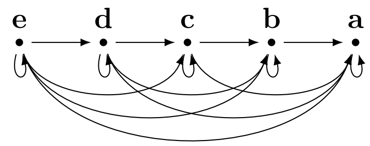


Figure 6: Diagram for a weak order R_1

Another example for an order is the 'less than or equal'-relation $\leq$ on the set of real numbers $\mathbb{R}$. This order is familiar form mathematics and it is represented graphically by the number line as in Figure 7.

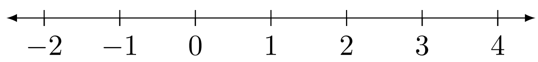


Figure 7: Number line for $\mathbb{R}$

The order $\preceq$ between individuals of a set A may be *represented* by means of numbers, usually the real numbers $\mathbb{R}$, that come with their own order $\leq$ (Krantz et al. 1971). The order $\preceq$ together with a set A forms a relational structure $\langle A, \preceq\rangle$. The numbers from $\mathbb{R}$ are assigned to the individuals in A in such a way that the empirical order between the individuals is reflected by the numerical order $\leq$ between those numbers. That is, there is a function φ that relates the relational structure $\langle A, \preceq\rangle$ to the structure $\langle \mathbb{R}, \leq\rangle$. This type of function defines different types of *measure functions* μ dependent on the property whose quantity is measured. We may measure happiness, temperature or length for example. Kennedy (2007) calls this dimension.

For our examples, we may construct the function $\mu_{happiness}$ such that it assigns the elements of A_1 numbers from $\mathbb{R}$: **e** the number 1, **d** the number 2 , **c** the number 3, **b** the number 4 and **a** the number 5. The happiest individual gets the highest number and the unhappiest the lowest and the order of the individuals according to the order R_1 is preserved in the order $\leq$ of the numbers assigned. μ relates $\langle A_1, R_1\rangle$ to the structure $\langle \mathbb{R}, \leq\rangle$.

The literature is not completely consistent in what is called a *scale*. Predominantly the function that maps the empirical relational structure into

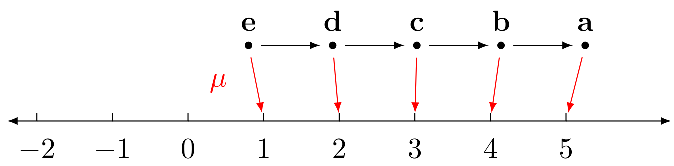


Figure 8: The measure function μ relating A_1 to $\mathbb{R}$

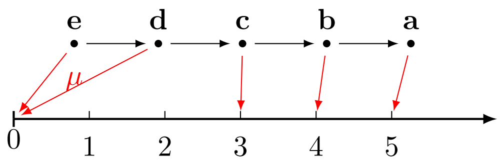


Figure 9: The measure function μ relating A_2 to $\mathbb{R}_0$

the set of real numbers is called a scale, i.e., φ (Krantz et al. 1971). Or the relational structure of reals together with the arithmetic relation $\leq$ and maybe a unit of measurement is called a scale, i.e. the relational structure that an ordered set of individuals is mapped to by means of a measure function (von Stechow 1984, 2009, Kennedy 2007, Solt 2019). Still others call the ensemble of the underlying structure, the measure function and the numerical representation a scale. We will follow the custom in behavioural sciences and use *property of an attribute* for the underlying relational structure (happiness, length, temperature, etc.), *measure function* for the function that assigns the numbers to individuals according to how much of the attribute they have ($\mu_{HAPPINESS}$, μ_{LENGTH}, $\mu_{TEMPERATURE}$, etc.) and *numerical representation* for the set of number assigned to the indiviudals measured.

If we measure the property of an attribute of a particular individual, we just read off the value from the respective measure function, i.e., a number that is assigned to that individual. A higher number is assigned to a happier/hotter/longer individual and a lower number to a less happy/hot/long one, for example. Measurement theory is concerned with the assumptions about the properties of an attribute such that measurement is possible. We just assume that these assumptions exist and we use them in our meta language in order to capture what the lingustic expressions mean.

6.4 Subtypes of scales

Behavioural sciences distinguish three types of scales (Krantz et al. 1971): Ordinal scales are the simplest type. Interval scales and ratio scales are dependent on additional properties of the relational ordering structure and the way the numbers are assigned.

Ordinal scales are constructed by just assigning the numbers to individuals as described above. The only requirement is that the empirical order is represented by the numerical order. The value assignment must be *order preserving*. The differences between the numbers assigned to the individuals do not represent measurable quantities and it is in principle difficult to calculate with the numbers because the spacing between the measures may differ (school grades, e.g.) from one category to the next. Ordinal scales, we typcially find in questionaires that investigate levels of happiness: "How happy are you on a scale from 1 to 5?". Somebody who belongs to the group with the number 2 on that questionaire scale is not 2 points less happy than somebody who belongs to the group assigned the number 4. Happiness is measured by assigning one the five numbers to the individuals in question. But in principle we are free in our choice of the number of answer options.

Interval scales are constructed by assigning fixed values to two reference individuals that mark two points on the scale [the interval]. Temperature, for example, is usually measured by means of an interval scale. The basic units, the degrees, do have names like Celsius or Fahrenheit. The Celsius scale, for example, is constructed by setting the boiling point of water as one reference point and the freezing point of water as the other reference point and the distance between those two points is devided by 100 into smaller intervals in order to fix additional reference points on the scale. We may calculate with degrees by adding or substracting them. Obviously, it can be 2 degrees colder on one day than on another day. Moreover, the values of those scales may range to negative numbers as can be observed on a very cold day in Chicago, for example. The value assignment is order preserving, as well.

The third type of scales, *ratio scales*, allows for the construction of a so-called *standard sequence* on the basis of a designated element (a base unit) of the set of individuals measured. In addition, to comparing individuals $\succeq$, it is possible to concatenate the individuals by the operation $\circ$. The relational structure is denoted as $\langle A, \succeq, \circ\rangle$.[13] The results of concatenation belong to the set of indiviudals, as well. If we choose a designated element a —the basic unit— from A and concatenate it with a perfect copy of a, we get a new individual a^* by $\circ$ that can undergo concatenation with a again and this procedure may be repeated again and again. The main point is, it is possible to count the basic units if concatenated and to state that two concatenated copies ($a \circ a'$) are twice the measure of one unit ($2a$), three are three times the measure of one unit, etc.. The sequence a, $2a$, $3a$, $\ldots$ is called a standard sequence. The concatenations of (copies) basic units may then be used in order to approximate other individuals. The numbers assigned to the individuals are *additive* with respect to concatenation: since if we approximate

[13]Krantz et al. suggest to think in rods that are compared with respect to length for getting an idea what $\succeq$ means. Moreover, the rods may be combined by laying them end to end in a straight line producing a kind of new rod and this procedure is called concatenation $\circ$. The length of the new rod depends on the length of the rods it is concatenated from.

one individual c with n copies of a and another individual b with n' concatenated copies of a then the concatenation of $b \circ c$ will be approximated by $n + n'$-many copies of a. With values of ratio scales multiplication and addition is possible.

Measuring an individual means counting copies of a basic unit using standard sequences that approximate the individual to be measured. The more fine grained the standard sequence the more precise the measurement. The spacing on the scale between the points depends on the choice of the basic unit and therefore does not differ.

The measure functions associated with a ratio scale are called *extensive measure functions*. The result of a concatenation is always comparable to the elements that are concatenated and those basic units are evidently smaller than the results of concatenating them. If it is possible to construct a so-called standard sequence of concatenations of individuals that are exact copies of each other, there are no negative numbers as values on the scale. Extensive measure functions are used in measuring weight, for example. If we add a 2 kg package to a 3 kg package, they weigh five kilo together. We use them to measure length. If we extend a 2 cm line by another 3 cm line we get a line of 5 cm. And so on. Ratio scales have no negative numbers as values. The values are the result of a counting process.

Sometimes a fourth type of scales *nominal scales* is considered, in addition. The result of sorting out individuals with respect to a property ("married" vs. "unmarried" for example) may be called nominal scale. If there is no ordering relation among the group, we may still assign numbers to the individual but the numbers do not reflect any ordering. But this means that a nominal scale is not a scale in the sense considered above. Constructing a scale always means a mapping of an order between individuals to an order of numbers that reflects the order of individuals.

In the next step, we consider the meaning of adjectives. The question is in what way an adjective's extension relates to scales and measure functions, respectively. It seems that the different types of scales relate to different types of adjectives. Nominal scales may be the basis for non-gradable adjectives, if they are scales at all, all the other types of scales are typical for gradable adjectives. Ordinal scales could prove relevant for evaluative adjectives like **happy**, interval scales for adjectives like **warm** that relate individuals to temperatures, and ratio scales for dimensional adjectives like **tall** and the like. That *scale structure* plays a role in adjectival semantics was shown in Kennedy & McNally (2005) and Kennedy (2007) for so-called absolute vs. relational scales.

6.5 Semantics of gradable adjectives

Measure functions are used in order to capture several aspects of language where numbers seem to play a role.[14] And the basic idea is that measure functions also are needed in order to interpret gradable adjectives.

We will be considered here with positive and negative gradable adjectives and introduce the proposal of Heim (2008) and von Stechow (2009) and its predecessors, most notably Cresswell (1976). The numerical representation of an ordering between individuals (i.e., the values of the measure function) is called *degrees* in linguistics. A measure function relates the individual to its measure (a degree). Since degrees represent an empirical order, they are ordered, as well. The empirical order on a set of individuals, i.e., quantifyable attributes like height, happiness or temperature and the like, is sometimes called a dimension (Kennedy 2007). The dimension restricts the measure function μ_{DIM}.

Measure functions may be represented by tables as in (51). By way of illustration, consider the potential ordering in the left column of the table and the values assigned to the individuals in the right column. The measure function is part of the metalanguage here. The individuals may have indicated how happy they are on a scale, as above. Where the least happy individual in the empirical order, Elba, gets the number 1 and the happiest individual, Amy, gets the number 5. The degrees of happiness depend on the situation where it is evaluated how happy an individual is. (51) is assumed to capture the same information that Figure 7 captures.

(51) $\mu_{\text{HAPPINESS}}(s) =$

Individual	Number
Elba	1
Dorit	2
Celine	3
Bernice	4
Amy	5

Heim (2008) and von Stechow (2009) assume that gradable adjectives relate individuals and degrees to give a truth value. In this sense, we can understand that the extension of a gradable adjectives is based on a scale.[15] The lexical entry for **happy** is the one in (52).

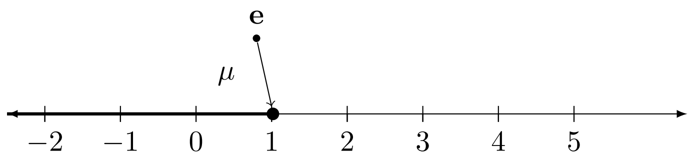
(52) $[\![\textbf{happy}]\!]^s = \lambda d.\lambda x. \vdash \mu_{\text{HAPPINESS}}(s)(x) \geq d \dashv$

[14] Cardinality may be seen as some kind of measure function, as well. It helps interpreting number words.

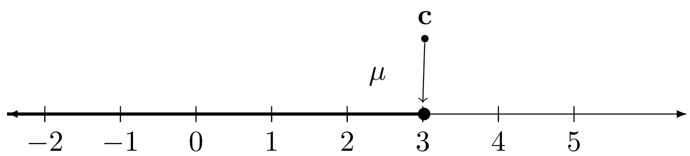
[15] There are several proposals in the literature how adjectives are interpreted compositionally. Kennedy (1999) elaborated the view that adjectives are just measure functions (and not relations). In this view, an adjective assigns to an individual a certain value from the set of reals. This view, that goes back to Bartsch & Vennemann (1972) and is evaluated in Heim (2001). The differences between the accounts do not matter for this summary.

The meaning of the adjective collects the values that the individual gets from the measure function *and all degrees below.* The set of degrees assigned to each individual x in our set of individuals may be captured as follows: $\downarrow \lambda d.\vdash \mu_{\text{HAPPINESS}}(s)(x) \geq d \dashv$ for each x. And it is convenient to visualize these sets of degrees as intervals on the number line related to the real numbers. Consider our setting in Figure 9 again on page 25: For the individual named Elba (**e**), it collects real numbers equal to or below 1 and for the individual named Celine (**c**), it collects real numbers equal to or below 3. The sets characterized by the functions in (53) and (54) are intervals since the carrier set, the set of real numbers is totally ordered.[16] The intervals have a highest element, the value of the measure function for the measured individual but no least element.

(53) $\downarrow \lambda d.\vdash \mu_{\text{HAPPINESS}}(s)(\text{Elba}) \geq d \dashv$
$= \{d \in \mathbb{R} \mid \mu_{\text{HAPPINESS}}(s)(\text{Elba}) \geq d\}$
$= (-\infty, 1]$



(54) $\downarrow \lambda d.\vdash \mu_{\text{HAPPINESS}}(s)(\text{Celine}) \geq d \dashv$
$= \{d \in \mathbb{R} \mid \mu_{\text{HAPPINESS}}(s)(\text{Celine}) \geq d\}$
$= (-\infty, 3]$



This view fascilitates the interpretation of the comparative and it is motivated by the interaction of comparative constructions with nominal quantifiers (Heim 2001).

6.6 The Comparative

The comparative may be expressed set theoretically as a relation between sets of degrees. In order to see this, we take our example again. Celine is happier than Elba in the setting in 9. And if it is true that Celine is happier than Elba, then the interval associated with Elba is a subset of the interval associated with Celine. A comparison between degrees amounts to a subset relation between sets of degrees.

[16] See Partee, ter Meulen & Wall (1990: p. 51) for total orders.

(55) $[\![\textbf{Celine is happier than Elba}]\!]^s$
$= \vdash \downarrow \lambda d_1.\vdash \mu_{\text{HAPPIN.}}(s)(\mathbf{e}) \geq d_1 \dashv \subset\downarrow \lambda d_2.\vdash \mu_{\text{HAPPIN.}}(s)(\mathbf{c}) \geq d_2 \dashv\dashv$
$= \vdash(-\infty, 1] \subset (-\infty, 3]\dashv$
$= 1$

By the method of abstraction, we may deduce the extension for the comparative morpheme. We have to spare the details here.

(56) $[\![\textbf{-er}]\!]^s = \lambda D_1.\lambda D_2.\vdash \downarrow D_1 \subset\downarrow D_2 \dashv$

The basic idea is that the comparative morpheme **-er** denotes a kind of quantificational determiner that is restricted by the **than**-clause. For interpretation the comparative morpheme and the adjective are not composed directly. The comparative morpheme and the **than**-clause together form a degree quantifier. The rest of the construction is the scope of that quantifier. Quantificational constructions in the realm of the semantics for adjectival constructions are tripartite analogous quantificational constructions in the realm of the semantics for nouns. The prerequisite of this view is that comparative constructions are elliptical. We basically follow the explanations in von Stechow (2009)and Heim (2008) that rest on Büring (2007b,a) how the truth conditions are derived, but we translate their view into the language of direct interpretation introduced in Zimmermann (2021). (57a) is the elliptical object language sentence. **c** abbreviates 'Celine' and **e** abbreviates 'Elba'.

(57) a. c is happier than e.
b. **c is happi-er than e ~~is happy~~**
c. **c is happi [-er than e is happy]**
d. **c is [-er than e is happy] happy**
e. **c is [-er than wh$_1$ e is t$_1$ happy] happy**

(57b) illustrates the intermediate step where the syntax relevant for interpretation is reconstructed, i.e. elliptical material is recovered. (57c) shows that the comparative morpheme and the **than**-clause are assumed to form a constituent. The comparative morpheme is an operator that appies to the extension denoted by the **than**-clause first. The **than**-clause and the comparative morpheme originate in the object position of a gradable adjective (57d). In addition, it has been argued that the than-clause actually patterns with relative clauses Chomsky (1977). This has lead to the assumption that **than**-clauses ARE interpreted with the same strategy relative clauses, see Heim (1985) in particular, and the interpretation procedure is standard since. A hidden **wh**-element is related to the a trace, called $\mathbf{t}_1$ here, in the argument position of the adjective, the degree argument (57e). The relation is called *binding* in syntax and signaled by an index **1** on the **wh**-element and its trace $\mathbf{t}_1$.[17]

As already mentioned, the comparative and the **than**-clause represent a quantifier. In order to be applicable to the rest of the sentence, i.e., its scope, some kind of grammatical rule is required analogous to what we find in the nominal domain where object quantifier constructions are interpreted by a special grammatical rule. The truth conditions in (55) maybe derived as in (58). In principle we follow the approach of Heim & Kratzer (1998) to the interpretation of quantificational constructions in Chapter 5 "Relative clauses" and Chapter 6–8 on quantifiers.

(58) $\llbracket \textbf{Celine is happier than Elba} \rrbracket^s$

$= \llbracket \textbf{c is [-er } \cancel{\textbf{than}} \textbf{ wh}_1 \textbf{ e is t}_1 \textbf{ happy] happy} \rrbracket^s$

$= \llbracket \textbf{[-er wh}_1 \textbf{ e is t}_1 \textbf{ happy]}_2 \textbf{ c is } t_2 \textbf{ happy} \rrbracket^s$

by Quantifier Raising (QR) or LF-movement

$= \llbracket \textbf{-er wh}_1 \textbf{ e is t}_1 \textbf{ happy} \rrbracket^s (\lambda d_2 . \llbracket \textbf{c is t}_2 \textbf{ happy} \rrbracket^{s,g[\mathbf{t_2} \to d_2]})$

by the Interpretation of QR, Predicate Abstraction

$= \llbracket \textbf{-er} \rrbracket^s (\llbracket \textbf{wh}_1 \textbf{ e is t}_1 \textbf{ happy} \rrbracket^s)(\lambda d_2 . \llbracket \textbf{c is t}_2 \textbf{ happy} \rrbracket^{s,g[\mathbf{t_2} \to d_2]})$

by the Interpretation of Quantifiers, Functional Application

$= \llbracket \textbf{-er} \rrbracket^s (\lambda d_1 . \llbracket \textbf{e is t}_1 \textbf{ happy} \rrbracket^{s,g[\mathbf{t_1} \to d_1]})(\lambda d_2 . \llbracket \textbf{c is t}_2 \textbf{ happy} \rrbracket^{s,g[\mathbf{t_2} \to d_2]})$

by the Interpretation of **Wh**-Constructions, Predicate Abstraction

$= \llbracket \textbf{-er} \rrbracket^s (\lambda d_1 . \llbracket \textbf{happy} \rrbracket^s (\llbracket \textbf{t}_1 \rrbracket^{s,g[\mathbf{t_1} \to d_1]})(\llbracket \textbf{e} \rrbracket^s))$

$\quad (\lambda d_2 . \llbracket \textbf{happy} \rrbracket^s (\llbracket \textbf{t}_2 \rrbracket^{s,g[\mathbf{t_2} \to d_2]})(\llbracket \textbf{c} \rrbracket^s))$

by Functional Application

$= \llbracket \textbf{-er} \rrbracket^s (\lambda d_1 . \llbracket \textbf{happy} \rrbracket^s (d_1)(\llbracket \textbf{e} \rrbracket^s))(\lambda d_2 . \llbracket \textbf{happy} \rrbracket^s (d_2)(\llbracket \textbf{c} \rrbracket^s))$

$= \vdash \downarrow \lambda d_1 . \vdash \mu_{\text{HAPPIN.}}(s)(e) \geq d_1 \dashv \subset \downarrow \lambda d_2 . \vdash \mu_{\text{HAPPIN.}}(s)(c) \geq d_2 \dashv \dashv$

In a first step, we used the sentence in its reconstructed form for interpretation as motivated in (57). **Than** is not interpreted and can be deleted in this account.[18]

In the second step, a syntactic operation takes place called quantifier raising (QR). Note that quantifier raising leaves a trace t_2. The trace is bound by the moved constituent [the comparative morpheme and the **than**-clause). The moved constituent is indexed, as well. The elements share the index. Indices are use to keep track which trace belongs to which moved element. And they play a role in the interpretation of syntactic constructions in the system of Heim & Kratzer (1998).

Quantifier raising targets all quantifers and can be understood as *type driven*. It derives the so-called Logical Form. The movement operation makes the syntax interpretable. It is in general not possible to combine a two-place predicate with a quantifier as its first argument. The quantifier

[17] See Bhatt & Pancheva (2004) and Alrenga & Kennedy (2014)f or a state of the art derivation of comparative sentences. The syntactic details were Bhatt & Pancheva deviate from our version are irrelevant here.

[18] Alrenga & Kennedy argue for an account with **than** interpreted instead of a hidden **wh** element. Again the arguments do not matter for the points made here.

is not of the right functional type. Quantifiers take a one-place predicate. We follow the proposal from the text book by Heim & Kratzer (1998) as already mentionned. But there are several possibilities to handle this problem (Zimmermann 2021: on the direct interpretation of object quantifiers from the nominal). Their method of interpretation is transfered to degree constructions here. The text books only handle nominal constructions. The general rule for composition of an indexed quantificational element can be stated as in (59).

D Q If α is a branching node consisting of an indexed quantifying constituent $[\beta]_i$, where i is a numerical index from $\mathbb{N}$, and the daughter γ, then for all $s \in LS$ and any variable assignment g the following holds:
$[\![\alpha]\!]^{s,g} = [\![\beta]\!]^{s,g}(\lambda x.[\![\gamma]\!]^{s,g[t_1 \to x]})$

Interpreting a quantification using quantifier raising has the same effects as interpreting it by a grammatical rule. The index that is attached to the moved constituent, the quantifier, i.e. **2**, can be used to introduce a lambda operator λd_2 on the quantifier's argument and the trace is designated to be translated by the same variable that the lambda operator consists of. The introduction of the lambda operator is usually called predicate abstraction. In addition, the interpretation function has to be modified for the traces to be interpreted correctly. This is accompished by adding an instruction for the interpretation of the trace to the interpretation function. The variables get their interpretation by a so-called variable assignment, a function that tells for every variable which value it gets in the suitable domain of interpretation. The variable assignment $g[\mathbf{t_2} \to d_2]$ ensures that the extension of the trace t_2 is the metavariable d_2. For traces, in general, we may use the following rule:

D T If α_i is a trace, where i is a numerical index from $\mathbb{N}$, then for all $s \in LS$ and any variable assignment g the following holds:
$[\![\alpha_i]\!]^{s,g} = g(\alpha_i)$.

In the third step, we split up the quantifier into the extension of the comparative morpheme and the extension of the **than**-clause.

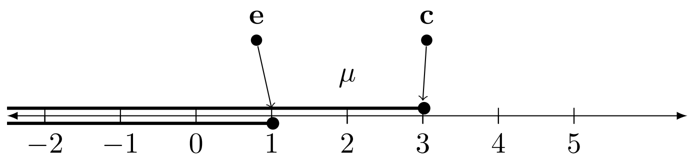
The **than**-clause is headed by a **wh**-element (indexed) and this signals that predicate abstraction may take place when interpreting the construction. The variable assignment $g[\mathbf{t_1} \to d_1]$ ensures that the extension of the trace t_1 is the metavariable d_1.

D W If α is a branching node consisting of an indexed wh-element constituent $[\mathbf{wh}]_i$, where i is a numerical index from $\mathbb{N}$, and the daughter γ, then for all $s \in LS$ and any variable assignment g the following holds:

$$[\![\alpha]\!]^{s,g} = \lambda x.[\![\gamma]\!]^{s,g[t_1 \to x]}$$

The result of this interpretation process is that the adjective extension may be applicable to a place holder degree (a scale point) and a subject. The interpretation of the **than**-clause and the matrix is completely parallel. Further steps just use our lexical entries for the adjective **happy**, the names **e** and **c** and the comparative morpheme **-er**. The relation between the intervals associated with Celine (c) and Elba (e) maybe illustrated with the diagram in Figure (59).

(59) Comparison of two intervals



This way of looking at comparative constructions has advantages when interpreting pairs of antonyms. If Celine is happier than Elba, it follows that Elba is unhappier than Celine. We would like to have a theory that derives this compositionally by just adding an extension for **un-**.

6.7 Negation and gradable adjectives

The extensions of gradable adjectives are relations that assign a truth value to a pair of indiviuals and degrees. Therefore, our earlier definition for affixal negation as stated in (40) cannot be used in order to combine directly with the extension of a gradable predicate. (40) can only be applied to one-place non-gradable adjecives. Applying it to a gradable predicate would result in a type-mismatch. But a variant of classical logical negation will do (see also von Stechow 2009)): We change the type of negation to a modifier of gradable adjectives as in (60). It is evident that negation in this definition for degree negation obeys the complementation hypothesis stated in (45) above. The basis for the application of this negation is classical logical negation.

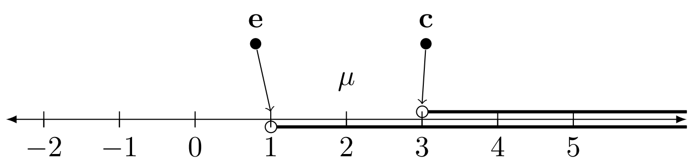
(60) $[\![\textbf{un-}]\!]^s_{\mathbf{D}} = \lambda R^{d(et)}.\lambda d.\lambda x.[\![\textbf{not}]\!]^s(R(d)(x))$

The example sentence **Elba is unhappier than Celine** gets the truth conditions in (61). The negation has the effect that the complement intervals are compared. In other words, the function of negation is to map a set of degrees to its complement set. The negation has narrow scope with respect to comparative morpheme. The gradable predicates negated by **un-** characterize the complement sets in $\mathbb{R}$ of the sets that the unnegated predicates characterize.

(61) $[\![$**Elba is unhappier than Celine**$]\!]^s$
$= [\![$**e is -er than c is un-happy un-happy**$]\!]^s$
$= \ldots$ (as above, see (58))
$= [\![$**-er**$]\!]^s\ (\lambda d_2.\ [\![$**un-happy**$]\!]^s(d_2)([\![$**c**$]\!]^s))\ (\lambda d_1.\ [\![$**un-happy**$]\!]^s(d_1)([\![$**e**$]\!]^s))$
$= \vdash \downarrow \lambda d_2.1 - \vdash \mu_{\text{HAPPIN.}}(s)(c) \geq d_2 \dashv \subset \downarrow \lambda d_1.1 - \vdash \mu_{\text{HAPPIN.}}(s)(e) \geq d_1 \dashv \dashv$
$= \vdash \downarrow \lambda d_2.\vdash \mu_{\text{HAPPIN.}}(s)(c) < d_2 \dashv \subset \downarrow \lambda d_1.\vdash \mu_{\text{HAPPIN.}}(s)(e) < d_1 \dashv \dashv$

The truth conditions are illustrated in (62). The thick lines represent the intervals associated with Celine (c) and Elba (e) respectively. These intervals are sometimes called neagtive extents. They cover the upper part of the numberline. Intervals that cover the lower part of the numberline are called *positive extents.*

(62) Comparison of two "negative" intervals



These intervals associated with the two individuals Celine and Elba are open on both sides. They end somewhere in infinity and they start right above the value that the measure function assigns to the individuals. The circles are use in this illustriation to signal openness of an interval. The measure of the indivudals is not part of the interval. Whereas negative intervals are open, positive intervals are half-closed.

Whether we say that Elba is unhappier than Celine or Celine is happier than Elba does not make a difference as far as the truth conditions are concerned. If there is a difference in meaning this might be an effect of a change in the subject. The unnegated sentence may be conceived as an information about Celine whereas the negated one containes information about Elba. But this is a difference in how information is packaged and not a difference in truth conditions. The idea to capture affixal negation on the basis of classical negation may capture this fact. The treatment of **un-** does not contradict the Complementation Hypothesis (HNEG). **un-** assigns a set of degrees its complement set of degrees.

There is, however, a third way of expressing the same facts. Heim (2008) is concerned with the question what a sentence like **Elba is less happy than Celine** means compared to the negated sentence in (61). Are there differences in meaning and of what kind is that difference? This is also a topic that Bierwisch (1989) deals with.

So far, we looked at constructions with adjectives in the comparative. Having seen the analysis of comparitives, it is maybe easier to understand that the positive form of gradable adjectives is comparative semantically, as well. Using an gradable adjective in the positive form expresses a comparison between a value of an individual to a standard value (what is a normal value, e.g.).

6.8 The Positive

Recall the diagram in Figure 6 before we check out the interpretation of the positive form of gradable adjectives. We dropped the arrows in Figure 10 that visualize transitivity and reflexivity. The diagram may stand for a line-up of five people, the elements of A_1 for example, where the two left-most people count as very unhappy (e) or rather unhappy (d) and the two right-most people as very happy (a) and quite happy (b). As before the people are related by the "is happer than (or equal)" relation R_1.[19] The expression **very unhappy** may label a class of people that are equally happy as Elba, the expression **rather unhappy** may label a class of people that are equally happy as Dorit, the expression **neither happy nor unhappy** may label a class of people that are equally happy as Celine, the expression **quite happy** may label a class of people that are equally happy as Bernice (b) and the expression **very happy** may label a class of people that are equally happy as Amy (a). We are familiar with such labels from questionnaires, a typical example for ordinal scales, see above. Bernice and Amy may count as happy and Dorit and Elba as unhappy. And Celine is in between.

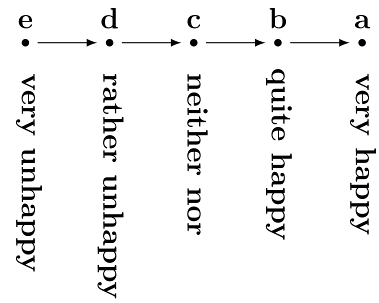
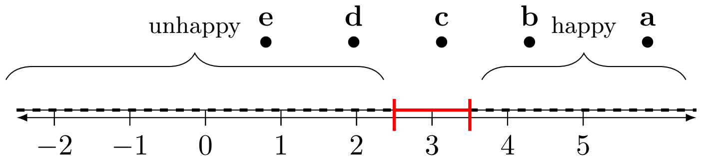


Figure 10: $\langle A_1, R_1 \rangle$ with labels for the classes

It seems possible to devide the numerical representation of the measured values into three subsets: the values that represent the degrees of happyness of the happy individuals, the values that represent unhappy individuals and the interval inbetween as illustrated in (63). Celine (c) would get a value in the middle interval in our setting.

[19] I think there is a differnce in how we construct the relational structure of the attribute with respect to who does the ranking. If a psychologist ranks the people may result in a different ranking than when the people rank themselves. Subjective meaning plays a role that we disregard at this moment.

(63) Three Parts: A neutral zone



One popular way of interpreting adjectives in the positive is the one by von Stechow (2009), Heim (2006), and Beck (2011). The positive is captured as an operator, as well, like the comparative. Von Stechow proposed the version in (64). The positive operator introduces a comparison between the actual value the measure function assigns to the indiviual that has the attribute and this middle interval. There is an additional function N that assigns the middle interval to an attribute like happiness. We abbreviate the attribute measured with the letter S, a relational structure. N and S are variables of the object language in this framework that get their values by a variable assignment. Which interval this is, is highly context dependent and there is quite an amount of interesting reaserach on the question where this interval has to be placed on the numerical representation (Lassiter & Goodman 0194). We cannot dive into this matter at this point and just presuppose that there is such a middle interval and that the context modelled by that variable assignment g supplies this interval.

(64) $[\![\textbf{POS-}_{\textbf{N,S}}]\!]^{s,g} = \lambda D.\vdash g(\mathbf{N})(g(\mathbf{S})) \subseteq\downarrow D\dashv$

The value that $g(N)$ assigns any relational structure $g(S)$ an interval form the set of real numbers $\mathbb{R}$. This interval is said to cover a neutral zone, some middle part of the numerical representation.

The sentence **Bernice is happy** is judged true in our scenario. This can be derived as in (65). The syntax involves quantifier raising again. Note that **POS-** is a universal quantifier: it relates two sets of degrees, just like **every** relates two sets of individuals. The positive operator is silent and recovered at LF (65b). In a second step, it is moved to a sentence initial position. Movement triggers coindexing of the moved element and its trace (65c).

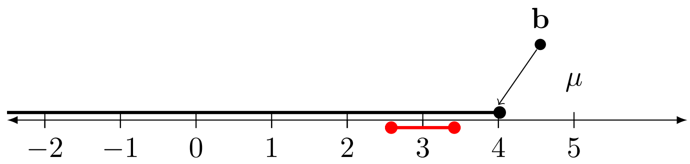
(65) a. Bernice is happy
b. **b is $\mathbf{POS_{N,S}}$ -happy**
c. **$\mathbf{[POS\text{-}_{N,S}]_1}$ b is $\mathbf{t_1}$ happy**

The truth conditions may be derived as in (66). Given that the neutral zone may be the interval covering Celine around the value 3 the sentence is true in our setting. The interval that is associated with Bernice covers the neutral zone completely.

(66) $[\![[\mathbf{POS}\text{-}_{\mathbf{N,S}}]_1 \text{ } \mathbf{b\ is\ t_1\ happy}]\!]^s$
$= [\![[\mathbf{POS}\text{-}_{\mathbf{N,S}}]]\!]^s(\lambda d_1.[\![\mathbf{b\ is\ t_1 happy}]\!]^{s,\mathbf{t_1} \to d_1})$
by Interpretation of Quantifier Raising
$= [\![[\mathbf{POS}\text{-}_{\mathbf{N,S}}]]\!]^s(\lambda d_1.\vdash \mu_{\text{HAPPINESS}}(s)(\text{b}) \geq d_1 \dashv)$
$= \vdash g(\mathbf{N})(g(\mathbf{S})) \subseteq \downarrow \lambda d_1.\vdash \mu_{\text{HAPPINESS}}(s)(\text{b}) \geq d_1 \dashv\dashv$
$= \vdash [2.5, 3.5] \subseteq (-\infty, 4] \dashv$
$= 1$

The relation between the neutral zone and the interval associated with Bernice may be illustrated as follows.

(67) True Positive: comparison to the neutral zone

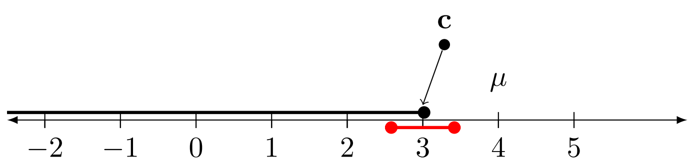


The interaction with sentential negation is predicted correctly in our scenario. **Celine is happy** is judged to be false because there are scale points in the numerical representation that are among the neutral zone but not covered by the interval associated with Celine (c). Consider (68).

(68) $[\![[\mathbf{POS}\text{-}_{\mathbf{N,S}}]_1 \text{ } \mathbf{c\ is\ t_1\ happy}]\!]^s$
$= [\![[\mathbf{POS}\text{-}_{\mathbf{N,S}}]]\!]^s(\lambda d_1.[\![\mathbf{c\ is\ t_1 happy}]\!]^{s,\mathbf{t_1} \to d_1})$
by Interpretation of Quantifier Raising
$= [\![[\mathbf{POS}\text{-}_{\mathbf{N,S}}]]\!]^s(\lambda d_1.\vdash \mu_{\text{HAPPINESS}}(s)(\text{c}) \geq d_1 \dashv)$
$= \vdash g(\mathbf{N})(g(\mathbf{S})) \subseteq \downarrow \lambda d_1.\vdash \mu_{\text{HAPPINESS}}(s)(\text{b}) \geq d_1 \dashv\dashv$
$= \vdash [2.5, 3.5] \subseteq (-\infty, 3] \dashv$
$= 0$

The illustration in (69) shows that the interval contributed the function g(N) applied to g(S) is not fully covered.

(69) False Positive: comparison to the neutral zone



Its negation **Celine is not happy** is therefore true. The derivation of the negative sentence is found in (70). As a sentential operator **not** negates the whole sentence. It has widest scope and in particular wide scope with respect to the positive operator. There is no scope interaction between the positive operator and negation.

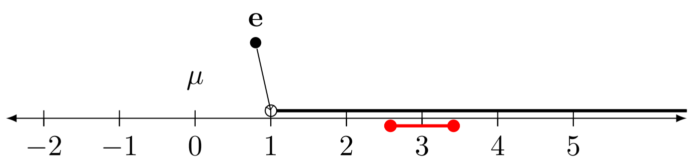
(70) $\llbracket \textbf{not } [\textbf{POS-}_{\textbf{N,S}}]_1 \textbf{ c is t}_1 \textbf{ happy} \rrbracket^s$
$= \llbracket \textbf{not} \rrbracket^s (\llbracket [\textbf{POS-}_{\textbf{N,S}}] \rrbracket^s (\lambda d_1 . \llbracket \textbf{c is t}_1 \textbf{happy} \rrbracket^{s, \mathbf{t_1} \to d_1}))$
$= \llbracket \textbf{not} \rrbracket^s (\llbracket [\textbf{POS-}_{\textbf{N,S}}] \rrbracket^s (\lambda d_1 . \vdash \mu_{\text{HAPPINESS}}(s)(\text{c}) \geq d_1 \dashv))$
$= 1 - \vdash g(\mathbf{N})(g(\mathbf{S})) \subseteq \downarrow \lambda d_1 . \vdash \mu_{\text{HAPPINESS}}(s)(\text{b}) \geq d_1 \dashv \dashv$
$= 1 - \vdash [2.5, 3.5] \subseteq (-\infty, 3] \dashv$
$= 1$

And the account carries over from positive-polar adjectives to negative-polar ones. Consider the sentence **Elba is unhappy**. The truth conditions may be derived as in (71).

(71) $\llbracket [\textbf{POS-}_{\textbf{N,S}}]_1 \textbf{ e is t}_1 \textbf{ un-happy} \rrbracket^s$
$= \llbracket [\textbf{POS-}_{\textbf{N,S}}] \rrbracket^s (\lambda d_1 . \llbracket \textbf{e is t}_1 \textbf{un-happy} \rrbracket^{s, \mathbf{t_1} \to d_1})$
by Interpretation of Quantifier Raising
$= \llbracket [\textbf{POS-}_{\textbf{N,S}}] \rrbracket^s (\lambda d_1 . \vdash \mu_{\text{HAPPINESS}}(s)(\text{c}) < d_1 \dashv)$
$= \vdash g(\mathbf{N})(g(\mathbf{S})) \subseteq \downarrow \lambda d_1 . \vdash \mu_{\text{HAPPINESS}}(s)(\text{b}) < d_1 \dashv \dashv$
$= \vdash [2.5, 3.5] \subseteq (1, \infty) \dashv$
$= 1$

The scenario may be illustrated as in (72). A negative interval, i.e., an interval that starts somewhere in the middle of the numerical representation and ends somewhere in infinity, is compared to the neutral zone. If it includes the neutral zone the sentence is true.

(72) A true positive with a "negative" interval



Double negation may be true of a sentence concerning Celine again. The extension of the sentence **Celine is not unhappy** may be represented as in (73).

(73) $\llbracket \textbf{not } [\textbf{POS-}_{\textbf{N,S}}]_1 \textbf{ c is t}_1 \textbf{ un-happy} \rrbracket^s$
$= \llbracket \textbf{not} \rrbracket^s (\llbracket [\textbf{POS-}_{\textbf{N,S}}] \rrbracket^s (\lambda d_1 . \llbracket \textbf{c is t}_1 \textbf{un-happy} \rrbracket^{s, \mathbf{t_1} \to d_1}))$
$= \llbracket \textbf{not} \rrbracket^s (\llbracket [\textbf{POS-}_{\textbf{N,S}}] \rrbracket^s (\lambda d_1 . \vdash \mu_{\text{HAPPINESS}}(s)(\text{c}) < d_1 \dashv))$
$= 1 - \vdash g(\mathbf{N})(g(\mathbf{S})) \subseteq \downarrow \lambda d_1 . \vdash \mu_{\text{HAPPINESS}}(s)(\text{b}) < d_1 \dashv \dashv$
$= 1 - \vdash [2.5, 3.5] \subseteq (3, \infty) \dashv$
$= 0$

It is obvious that negation does not cancel out in this system. The positive

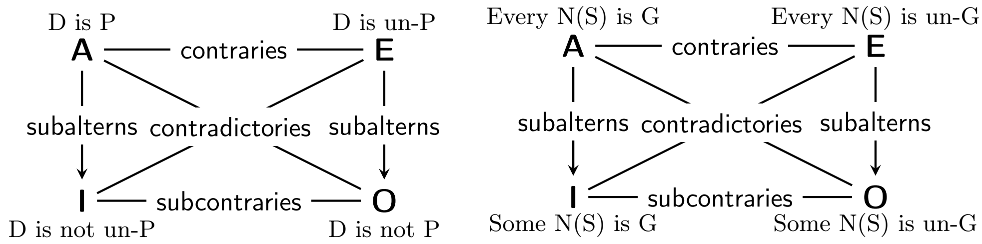


Figure 11: Square of opposition: Degree predication

operator intervenes between the two types of negation: the affixal negation **un-** and the sentential negation **not**. Therefore, the sentence in (74) is true in our scenario.[20]

(74) Celine is neither happy nor unhappy.

The square of opposition for comparisons on the basis of positives may be visualized as in Figure 11. The figure on the left represents the version of Horn (1989). The figure on the right takes the contribution of the POS-operator as a universal quantifier over degrees seriously. In this case G represents any a function that charactizes the set of degrees D has. gradable predicate independent on a positive operator. The positive operator introduces quantification over degrees. And $N(S)$ is meant to represent the degrees in the neutral zone. Gradable adjectives fit nicely into the picture of quantification.

So far, we only considered so-called relative adjectives. As already mentioned above, it is less clear how this account is transferred to pairs of absolute adjectives like **clean** vs. **dirty**. What is called a neutral zone in the account presented may be different from adjective class to adjective class dependent on the type of scale they associate with. And a different version of the square of opposition is used in Gotzner, Solt & Benz (2018). They investigate the relation between elements of quadruplets like ⟨**brilliant**$_A$, **intelligent**$_I$, **not intelligent**$_E$, **not brilliant**$_O$⟩ and ignore the contribution of the positive. It might well be that an intensifier like **very** is part of the meaning here (instead of the hidden positive operator), where **brilliant** is a lexicalization of **very intelligent**.

And an open question remains how the sentence **Maria ist nicht unstolz auf ihren Sohn** is interpreted. The prediction is that Mary's level of proudness is either in the neutral zone or above. But litotes has an pragmatic effect, called negative strengthening (Horn 2017, Krifka 2007).

[20] There is one "problem" with this account. If the neutral zone shrinks down to a point, say the exact measure of Celine on the happiness-scale by coincidence, then Celine is neither happy nor unhappy becomes contradictory. This problem might be cicumvented by a restriction on the neutral zone, not to be an interval of point size. Or we engage in a pragmatic explanation why (i) may be true (Krifka 2007).

6.9 . . . and implicit negation?

Pairs of antonyms are often lexicalized differently in the sense that they do not share a root. Whereas **unhappy** is derived from **happy** by attaching **un-**, there is no obvious connection between **big** and **small** or **happy** and **sad**. There is however research in the psycholinguistic domain that shows that the negative polar, more marked elements of the pair pattern with negative phenomena (Clark & Chase 1972, Clark 1974: for example), in general, and this observation may be an argument in favor of treating implicitly negative adjectives on a par with explicitly negative adjectives.

Antonymy then boils down to the following relation for pairs of antonyms like **big/small** and **happy/sad**.

(75) a. $[\![\textbf{small}]\!]^s = [\![\textbf{un-}_{\textbf{D}}]\!]^s([\![\textbf{big}]\!]^s)$
b. $[\![\textbf{sad}]\!]^s = [\![\textbf{un-}_{\textbf{D}}]\!]^s([\![\textbf{happy}]\!]^s)$

This equivalence in meaning is a hypothesis and subject to further investigation. Heim (2008) and Büring (2007a,b) discuss arguments for decomposition of those antonyms.

7 Conclusion

The account presented answers the questions introduced above. We present the arguments for non-classical negation and show that these arguments are not valid.

The argument from double negation In classical logic double negation cancels out but in combination with gradable adjectives this effect is not observed. This could be an argument in favor of non-classical negation (Horn & Wansing 2020). Therefore, two types of negation, contrary and contradictory negation, are semantically relevant.

We observe that all these arguments only concern constructions that are comparative in nature. If gradable adjectives are interpreted according to the account of degree semantics (von Stechow 2009) presented as relations between individuals and degrees, negation may have the effect of assigning a complement set of degrees to the set of degrees an individual is associated with by an unnegated gradable adjective. The square of opposition does not collaps with gradable predicates. $\langle$**POS-happy**$_A$,**not POS-unhappy**$_I$,**POS-unhappy**$_E$, **not POS-happy**$_O\rangle$ form a quadruplet in the sense of Horn. We do not need two different types of negation. That double negation does not cancel out is an *intervention effect of the positive operator* that is used in order to interpret gradable adjectives, in general.

This shows that Horn's argumentation that contrary negation (i.e. non-classical) is witnessed by the fact that double negation does not cancel out in combination with gradable adjectives does not hold up.

The argument from scope The negation with **un-** may result in a stronger meaning than the negation with **not** even if there is no scopal interaction with negation and nominal quantifiers. Jacobs (1991) discussed a difference in meaning between **nicht gebildet** 'not educated' and **ungebildet** 'uneducated' and proposed the notions strong negation (un-) and weak negation, where only weak negation corresponds to logical negation.

This difference may be captured by a difference in scope nevertheless. Negation interacts with a (hidden) positive operator that comes with the gradable adjective in these examples, as well. In **nicht gebildet** 'not educated', negation has wide scope with respect to the positive operator that is part of the adjective and quantificational. In **ungebildet** 'uneducated', **un-** (in the degree negation variant) **un-**$_{\mathbf{D}}$ has narrow scope with respect to the positive operator. This difference nurtures the difference of strength and might still be classified as a *difference in scope with respect to the positive operator*. The order of interpretation of negation and the type of modifier (modifying degree predicates or not) may play a significant role here. This shows that Jacobs's argumentation that stronger negation (i.e. non-classical) is witnessed by the fact that differences in meaning even occur if no quantifiers are involved in the construction does not hold up. Degree quantifiers may be silent. The cases of strong negation are actually cases of universal quantification over degrees, The cases of weak negation are actually cases of existential quantification over degrees. And the two types of construction stand in the sense relation of implication, i.e., weak negation is a subaltern of strong negation.

In general, it seems that contrary negation or scale reversal is not needed in order to account for the contraries expressed by predications with gradable adjectives. The main point is that constructions with gradable adjectives are in fact quantifications, but the elements quantified over are degrees (points on a scale) not individuals. This view does not exclude that there are pragmatic inferences that lead to a strengthened reading of a negative expression (Gotzner, Solt & Benz 2018, Mazzarella & Gotzner 2021).

Therefore, the complementation hypothesis still seems valid. The main two alleged counterexamples concern gradable predicates and they could be eliminated by a more fine-grained interpretation of gradable adjectives. In the case of sentential negation the argument of negation is (a function that characterizes) a proposition (a set of situations). In the case of non-gradable predicates, negation operates on a function that characterizes a set of individuals and amounts to complementation. And in the case of gradable predicates it operates on a degree predicate. Negation then just assigns this predicate the complement set of degrees. If there are differences in meaning between sentential negation and the other types of negation, it is *always* the result of scopal differences. Negation interacts with nominal quantifiers and with degree operators.

References

Alrenga, Peter & Christopher Kennedy. 2014. *No more* shall we part: Quantifiers in English comparatives. *Natural Language Semantics* 22(1). 1–53. https://doi.org/10.1007/s11050-013-9099-4. https://doi.org/10.1007/s11050-013-9099-4.

Bartsch, Renate & Theo Vennemann. 1972. Relative adjectives and comparison. In P. Schachter & G. Bedell (eds.), *Ucla papers in syntax*, vol. 2. Los Angeles: University of California Press.

Beaver, David & Henk Zeevat. 2007. Accommodation. In Ramchand G. & C. Reiss (eds.), *The Oxford Handbook of Linguistic Interfacesinterfaces,* , 533–538. Oxford University Press.

Beck, Sigrid. 2011. Comparison Constructions. In Klaus von Heusinger, Claudia Maienborn & Paul Portner (eds.), *Semantics. An International Handbook of Natural Language Meaning*, vol. 2, 1341–1389. Berlin: De Gruyter.

Bhatt, Rajesh & Roumyana Pancheva. 2004. Late Merger of Degree Clauses. *Linguistic Inquiry* 35(1). 1–45.

Bierwisch, Manfred. 1989. The semantics of gradation. In Manfred Bierwisch & Ewald Lang (eds.), *Dimensional adjectives: grammatical structure and conceptual interpretation*, 71–261. Berlin: Springer-Verlag.

Büring, Daniel. 2003. On D-trees, beans, and B-accents. *Linguistics and Philosophy* 26. 511–545. https://doi.org/10.1023/A:1025887707652.

Büring, Daniel. 2007a. Cross-Polar Nomalies. *Proceedings of SALT 17*.

Büring, Daniel. 2007b. More or less. *Proceedings of CLS 43*.

Chomsky, Noam. 1977. On WH-movment. In Peter Culicover, A. Akmajian & Tom Wasow (eds.), *Formal syntax*, 71–132. Academic Press.

Clark, Herbert. 1974. Semantics and comprehension. In Thomas Seboek (ed.), *Current trends in lingusitics*. The Hague: Mouton.

Clark, Herbert & William Chase. 1972. On the process of comparing sentences against pictures. *Cognitive Psychology* 3. 472–517.

Cresswell, Maxwell J. 1976. The semantics of degree. In Barbara Partee (ed.), *Montague grammar*, 261–292. New York: Academic Press.

Cresswell, Maxwell J. 1982. The autonomy of semantics. In Stanley Peters & Esa Saarinen (eds.), *Processes, Beliefs, and Questions*, 69–86. Dordrecht: Reidel.

Frege, Gottlob. 1892. Über Sinn und Bedeutung. *Zeitschrift für Philosophie und philosophische Kritik* NF 100. 25–50.

Gotzner, Nicole, Stephanie Solt & Anton Benz. 2018. Scalar diversity, negative strengthening, and adjectival semantics. *Frontiers in Psychology*

9. 1659. `https://doi.org/10.3389/fpsyg.2018.01659`. `https://www.frontiersin.org/article/10.3389/fpsyg.2018.01659`.

Hajičová, Eva. 1994. Topic, focus, and negation. In Peter Bosch & Robert van der Sandt (eds.), *Focus and natural language processing*, vol. 2. IBM Germany.

Heim, Irene. 1985. Notes on Comparatives and Related Matters.

Heim, Irene. 2001. Degree operators and scope. In Caroline Féry & Wolfgang Sternefeld (eds.), *Audiatur vox sapientiae*, 214–239. Berlin: Akademie Verlag. `http://www.sfs.uni-tuebingen.de/~astechow/Festschrift/Heim-7-6-komplett%20fertig.pdf`.

Heim, Irene. 2006. *Little*. In Masayuki Gibson & Jonathan Howell (eds.), *Proceedings of salt xvi*, 35–58. Ithaka, NY: CLC, Cornell Univ.

Heim, Irene. 2008. Decomposing antonyms. *Proceedings of Sinn und Bedeutung* 12(212-225).

Heim, Irene. 1986/87. *Presupposition Projection.* Tech. rep. University of Texas at Austin.

Heim, Irene & Angelika Kratzer. 1998. *Semantics in Generative Grammar.* Oxford: Blackwell.

Horn, Laurence R. 1989. *A natural history of negation.* Reissue edition 2001. With a forew. by Laurence R. Horn. Stanford, CA: CSLI.

Horn, Laurence R. 2017. Lie-toe-tease: double negatives and unexcluded middles. *Philosophical Studies* 174(1). 79–103. `https://doi.org/10.1007/s11098-015-0509-y`.

Horn, Laurence R. & Heinrich Wansing. 2020. Negation. In Edward N. Zalta (ed.), *The Stanford encyclopedia of philosophy*, Spring 2020. Metaphysics Research Lab, Stanford University.

Jacobs, Joachim. 1991. Negation. In Arnim von Stechow & Dieter Wunderlich (eds.), *Semantik/Semantics*, 560–596. Berlin: de Gruyter.

Kennedy, Christopher. 1999. *Projecting the adjective: The syntax and semantics of gradability and comparison.* New York: Garland.

Kennedy, Christopher. 2007. Vagueness and grammar: the semantic of relative an d absoute gradable. *Linguistics and Philosophy* 30(1). 1–45.

Kennedy, Christopher & Louise McNally. 2005. Scale Structure, Degree Modification, and the Semantics of Gradable Predicates. *Language* 81(2). 345–381. `https://doi.org/10.1353/lan.2005.0071`. `http://dingo.sbs.arizona.edu/~hharley/courses/PDF/KennedyMcNallyScales.pdf`.

Krantz, David H., R. Duncan Luce, Patrick Suppes & Amos Tversky. 1971. *Foundations of Measurement.* Vol. I. New York/London: Academic Press, Inc.

Krifka, Manfred. 2007. Negated antonyms: creating and filling the gap. In Uli Sauerland & Penka Stateva (eds.), *Presupposition and Implicature in Compositional Semantics*, 163–177. Houndmills: Palgrave Macmillan.

Lassiter, Daniel & Noah D. Goodman. 0194. Adjectival vagueness in a bayesian model of interpretation. *Synthese* 10. 1–36.

Löbner, Sebastian. 1999. *Wahr neben falsch*. Tübingen: Niemeyer.

Lyons, John. 1977. *Semantics*. Vol. 1. Cambridge University Press. `https://doi.org/10.1017/CBO9781139165693`.

Mazzarella, Diana & Nicole Gotzner. 2021. The polarity asymmetry of negative strengthening: dissociating adjectival polarity from facethreatening potential. *Glossa: a journal of general linguistics* 6(1). 47 pages.

Meier, Cécile & Inga Kohlhoff. 1997. *Presupposition and Information Structure*. Tech. rep. Universität Tübingen.

Parsons, Terence. 2021. The Traditional Square of Opposition. In Edward N. Zalta (ed.), *The Stanford encyclopedia of philosophy*, Fall 2021. Metaphysics Research Lab, Stanford University.

Partee, Barbara, Alice ter Meulen & Robert Wall. 1990. *Mathematical Methods in Linguistics*. Dordrecht.

Partee, Barbara H. & Mats Rooth. 1983. Generalized conjunction and type ambiguity. In Rainer Bäuerle, Christoph Schwarze & Arnim von Stechow (eds.), *Meaning, use, and interpretation of language*, 361–383. Berlin & New York: de Gruyter.

Russell, Bertrand. 1905. On denoting. *Mind* 14(56). 479–493.

Sapir, Edward. 1944. Grading: a study in semantics. *Philosophy of Science* 11. 93–116.

Solt, Stephanie. 2019. Adjective meaning and scales. In Chris Cummins & Napoleon Katsos (eds.), *The Oxford Handbook of Experimental Semantics and Pragmatics*. Oxford University Press.

von Stechow, Arnim. 1984. Comparing semantic theories of comparison. *Journal of Semantics* 3. 1–77.

von Stechow, Arnim. 2009. The temporal degree adjectives *früher/später* and the semantics of the positive. In Monika Rathert & Anastasia Giannakidou (eds.), *Quantification, Definiteness, and Nominalization*. Oxford: Oxford University Press.

Storjohann, Petra. 2015. *Deutsche antonyme aus korpuslinguistischer sicht – muster und funktionen*. Mannheim: Institut für Deutsche Sprache.

Strawson, Peter F. 1950. On Referring. *Mind* 235. 320–344.

Weicker, Merle & Petra Schulz. 2020. Not all gradable adjectives are vague. experimental evidence from adults and children. *Sinn und Bedeutung* 24(3). 406–422.

Wittgenstein, Ludwig. 1959. *Tractatus logico-philosophicus Logisch-philosophische Abhandlung*. Oxford: Blackwell.

Zimmermann, Thomas Ede. 2021. Semantics I-III. ms: Goethe University.

Zimmermann, Thomas Ede & Wolfgang Sternefeld. 2013. *Introduction to Semantics. An Essential Guide to the Composition of Meaning*. Berlin, Boston: De Gruyter Mouton.