Antonyms and adjectival Horn Scales

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Abstract

Horn discusses the longstanding question of how many types of negation there are. And quite some researchers subscribe with him to the hypothesis that there is more to negation than its function in classical logic. In this paper, I discuss three phenomena that seem to support this view: (i) there is a difference between weak and strong negation, though weak negation may be strengthened to strong negation, (ii) double negation often does not cancel out and (iii) the Square of Opposition does not readily generalize to adjectival Horn Scales. The three phenomena concern constructions with scalar adjectives. I demonstrate (following Heim and von Stechow) that a theory of gradability where scalar adjectives relate individuals and degrees and positive morphology is associated with a silent degree quantifier explains the phenomena. Strong negation is classical logical negation with narrow scope with respect to the degree quantifier, and spelled out as affixal negation, for example. Pairs of antonyms are linked by strong negation (Heim). Weak negation is also classical logical negation and has wide scope with respect to the degree quantifier. It is spelled out by sentential negation. Negative strengthening is related to degree quantifier movement across sentential negation and turns out to be a scope effect. Double negation does not cancel out because the degree quantifier intervenes between the negations. Von Stechow captures the degree quantifier as a universal quantifier that quantifies over degrees from the Zone of Indifference — the extension gap, that is characteristic for antonyms. I show that pairing up the Square of Opposition with adjectival Horn Scales, necessitates a degree quantifier with existential force. The degree quantifier, therefore, turns out to be force variable, like other quantificational expressions in the nominal and modal domain. Weakening the quantificational force of the degree quantifier from a universal to an existential is triggered by widening the Zone of Indifference.

Contents

1	Introduction 1			
	1.1 Negation is Type-Flexible	1		
	1.2 Strong and weak readings	2		
2	Three puzzles			
	2.1 Strong and weak negation	5		
	2.2 Double negation	6		
	2.3 Adjectival Horn Scales	9		
3	'Strong' negation is truth functional	10		
	3.1 Semantics of scalar adjectives	10		
	3.2 Strong and weak negation	12		
	3.3 Negative Strengthening: a scope effect?	15		
	3.4 The Semantics of Double Negation	16		
4	Sense relations across different types of domains	19		
	4.1 Singular and Traditional Square of Opposition are family	19		
	4.2 Two word fields for <i>possible</i>	21		
5	Variation in the 'Zone of Indifference'	23		
	5.1 Adding Extremeness	23		
	5.2 The Dual of the Positive Operator	25		
	5.3 Negating the existential: a form of negative strengthening \therefore	27		
	5.4 Towards Attenuation	28		
	5.5 Adjectival Horn Scales and the Singular Square of opposition	30		
	5.6 Attenuation and understatement	31		
6	Polarity Asymmetries			
7	Conclusion	37		
8	Acknowledments	38		

1 Introduction

Negative polar adjectives come in at least two varieties in English and German: either negated by an affix such as the prefix **un-** as in **unhappy**, or they have an implicit negative meaning component as in **sad**, for example. The main questions in this paper are long-standing: What is the function of adjectival negation with the affix un- and adjectival implicit negation? And how does adjectival negation relate to sentential negation? The investigation may shed light on antonymy: how antonyms are analyzed semantically and how pairs of antonyms like happy and unhappy or happy and sad relate to each other and their negated counter parts like **not happy**, **not** unhappy and not sad. Antonymy is captured as a relation between a pair of words. In a sentence, the elements of the pair may be substituted by each other and the propositions that the resulting two sentences express stand in the sense relation of *contradiction* or *contrariety* (Lyons 1977).¹ And, it helps understand how pairs of elements of adjectival Horn Scales like **con**tent and happy or doubly negated not unhappy and happy, for example, relate. Horn Scales are based on an *entailment relation* between at least two propositions expressed by sentences where just one element of the Horn scale is substituted for the other. Contrariety, contradiction and entailment are basic logical relations, mediated by negation, and usually visualized by the Square of Opposition. But negation may turn up in many different positions and guises in syntax. This fact is captured in terms of type flexibility.

1.1 Negation is Type-Flexible

Jacobs (1991: 569, my translation) states the following hypothesis in his overview on negation: We may call this hypothesis the *Negation Hypothesis* (HNEG).

(1) For every natural language L it holds that in an adequate semantic theory, every occurrence of negation is representable with NEG.

This is a hypothesis about meaning and does not exclude that neg-words like **nobody** do not associate with NEG as has been argued for Negative Concord languages if they are in the context of another negative marker that DOES associate with NEG (Herburger 2001, van der Wouden & Zwarts 1993, Weiß 1999, Zeijlstra 2004, Penka 2011). It is a hypothesis about the function of NEG. One question is whether HNEG is true or whether there are counterexamples that are not possible to be explained away.

NEG may be taken to represent the extension of sentential negation, the meaning of **not** in English. In indirect interpretation, **not** is usually translated as \neg .

¹For criteria for the difference between negative polar and positive polar adjectives, I refer to Ruytenbeek, Verheyen & Spector (2017: 10–11).

(2) $|\mathbf{not}| = \neg$

Its semantics corresponds to the semantics of classical logical negation (truthvalue reversal) as defined in (3).

(3) $\llbracket \neg \rrbracket$ = the function f such that for any truth value t from the set of truth values T (= {1,0}) the following holds: $f(t) = \begin{cases} 1 & \text{if } t = 0 \\ 0 & \text{if } t = 1 \end{cases}$

If (3) is abbreviated with a lambda term for functions, we arrive at the semantics in (4). Truth-value reversal is captured arithmetically. The negation operator is of type (tt) and applies to sentence extensions, type t, in this case.

(4) NEG = $\llbracket \neg \rrbracket = \lambda t.1 - t$

Predicate negation and verb negation may be captured on the basis of this definition rendering negation a type-flexible operator (Keenan & Faltz 1985, Zwarts 1991), as argued for in Categorial Grammar. Similarly, other variants may be defined for all *n*-place predicates, i.e. different types of verb negation. (5a) applies to intransitive verbs and verb phrases, both of type (et). (5b) applies to transitive verbs. (5c) to quantifiers as in **not everybody**. But, all are translated on the basis of \neg , obeying HNEG.

(5) a.
$$|\operatorname{not}_{IV}| = \lambda P^{et} \cdot \lambda x^e \cdot \neg (P(x))$$

b. $|\operatorname{not}_{TV}| = \lambda P^{e(et)} \cdot \lambda y^e \cdot \lambda x \cdot \neg (P(y)(x))$
c. $|\operatorname{not}_Q| = \lambda P^{(et)t} \cdot \lambda Q^{et} \cdot \neg (Q(P))$

The extension of the predicate **[not married]**, for example, represents a function that *characterizes* the set of individuals that are not married — which is the same as *the complement* of the set of individuals that are married (with respect to the universe of all individuals). This follows under the assumption of text book variants for the extension of **married** as a 1-place predicate and how compositionality is captured in formal semantics, namely by functional application. Negation may modify sentences, predicates, verbs, etc.. It's type flexibility represents (in part at least) the surface positional flexibility in natural language.

1.2 Strong and weak readings

Negation also interacts with quantifiers in allowing scope ambiguities. It is interesting to note in our respect that sentential and affixal negation show different scopal behavior. Whereas sentential negation **not** interacts with nominal quantifiers in English (and German), affixal negation (or implicit negation) does not, as exemplified in (6) and (7).

(6)	All politicians are not married.	Scope ambiguity
	a. 'it is not the case that all politicians are marriedb. 'all politicians are such that they are not married	
(7)	All politicians are unmarried No	scope ambiguity
	a. #'it is not the case that all politicians are married	ed' weak
	b. 'all politicians are such that they are not married	ied' strong

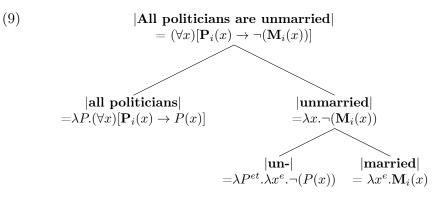
The negative expression **not** in (6) may either take wide scope with respect to the quantifier **all politicians** and is interpreted as sentential negation (6a), type (tt), or it takes narrow scope with respect to **all politicians** in English and may be interpreted as predicate negation as in (6b), type (et)(et). The first reading is termed the weak reading, the latter the strong reading, and intonation [a rising tone on the quantifier and a falling one negation] may resolve the ambiguity (Büring 1997). The strong reading entails the weak reading. The weak reading is considered a case of scope inversion in the literature. But there is no difference in the interpretation of negation as NEG and there is no difference in the position of the negative marker **not** at the surface.

Affixal negation **un-** in (7) resolves the ambiguity. Example (7) only has the strong reading, the one where quantifier and negation are read in the same order as they appear in the string of words. *Word internal negation* may not take scope with respect to word external quantifiers. It is internal negation in a morphological sense. Intonation cannot trigger a difference in the readings.

This behavior of affixal negation can be derived by assuming that **un**only expresses predicate negation and is always of a more complex type than (tt): i.e. type (et)(et) as in (8), for example.

(8) $|\mathbf{un}_{adj}| = \lambda P^{et} \cdot \lambda x^e \cdot \neg (P(x))$

The formalism used in this paper builds upon common practice in formal semantics. It is a variant of indirect interpretation. The analysis is kept as simple as possible. With (8) we may derive the representation of (7b) across the morpho-syntactic boundary, as illustrated in (9).



In addition to the semantic argument from scopal issues, there is a semanticopragmatic argument that the scope of affixal negation is confined to the words whose meaning it modifies (Horn 1989: 23f, 103). Whereas sentential negation allows for presupposition cancellation (10a), it is impossible to observe presupposition cancellation with affixal negation (10b). Again, affixes are bound morphemes and their scope is bound to the word.²

(10) a. The king of France is not married. There is no king of France.b. The king of France is unmarried. #There is no king of France.

The Negation Hypothesis (HNEG), that negative markers always relate to NEG, has been called into question, however. The aim of the paper is to show that this conclusion is not necessary. I defend the hypothesis that all cases of alleged illogical negation may be traced back to scope interaction of an adjectival quantifier — an element that quantifies over degrees — and classical negation.

The paper is organized as follows: In Section 2, I bring forward three old puzzles that seem to show that affixal negation (or implicit negation as in **sad**) is different in meaning form classical negation. In Section 3, I introduce the semantics of scalar adjectives. The main assumption is that adjectives in the positive come with a (hidden) universal adjectival quantifier — the POSITIVE Operator — that may take wide scope or narrow scope with respect to sentence negation. This assumption comes from von Stechow (2009a) and it may explain the differences in meaning. For the definition of

²This argument here is about definite descriptions like the **the King of France**. The fact that sentential negation can cancel presuppositions of subjects even if in predicate position is also discussed with respect to proper names by Horn (1989) (interpreted like definite descriptions) and there is considerable discussion in the philosophical literature whether predicate negation in the form of **not** may be able to cancel alleged existential presuppositions of subject quantifiers (see Parsons 2021: for discussion): it can. Dolf Rami (2024, p.c.) discusses the history of strong and weak negation and its effects on the definedness of truth conditions in his presentation "Negation and Existence": the negative existential sentence **Sandy Island is inexistent** is a notable counterexample to the observation here because affixal negation CAN be taken to cancel the existential import of a proper name in this case.

adjectival un- in combination with degree predicates, I refer to Heim's negation theory of antonymy (Heim 2008). Her adjectival negation is a version of classical negation and it's definition obeys HNEG, but it seems scopally inert. It shows up as the realization of the negative affix **un-** as in **unhappy** or realizes implicit negation as in sad. What is less clear is how intermediate expressions like **content** fit the picture and whether there could be a difference in meaning between sad and unhappy. In Section 4, I show that this assumption nicely accounts for sense relations between sentences with pairs of antonyms and their negation in terms of a difference in scope. The Square of Opposition may be built up on the basis of adjectival antonyms and sentential negation. The observation is that pairs of antonymous adjectives pattern with so called contradual operators (like every N and no N, or must and must not). Double negation as in not unhappy does not cancel out because the universal degree quantifier intervenes between the two occurrences of negation. And Section 5 is concerned with adjectival Horn Scales. The point of this section is to argue that the adjectival universal quantifier (normally a universal quantifier) may have an existential counterpart in Horn Scale contexts. Adjectives with positive morphology may be force variable like other types of quantifiers from other domains. It will be shown that the existential positive operator is triggered by widening the domain of quantification of the (universal) degree quantifier. This then explains how the Square of opposition and measurement scales relate. The last section 6 applies the explanation to experimental findings.

2 Three puzzles

I discuss three phenomena that seem to support the view that there is negation that does not behave logically: (i) there is a difference between weak and strong negation, though weak negation may be strengthened to strong negation, (ii) double negation often does not cancel out and (iii) the Square of Opposition does not readily generalize to adjectival Horn Scales.

2.1 Strong and weak negation

Jacobs (1991: p. 593) discusses the following difference in meaning between (11) and (12). Als phrases are scope islands in German. And the same is true for infinitival constructions in English. There is obviously no presuppositional scope effect involved between the subject phrase **the king of France** and negation. Still, it seems that there is a difference in meaning between sentential negation and affixal negation. This is Jacobs' puzzle. Again, we observe a stronger meaning and a more weak reading comparable to the scope effects above. The Negation Hypothesis seems only to explain the weak reading, though. Since there is a difference in readings that may only originate with negation, we may doub the two types of negation strong and weak negation.

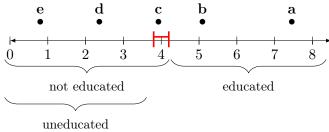
- (11) **Der König von Frankreich erwies sich als nicht gebildet.** 'The king of France turned out not to be educated.' weak
- (12) Der König von Frankreich erwies sich als ungebildet.
 'The king of France turned out to be uneducated.' strong

It seems mandatory to discuss a meaning of the negative affix **un-** that is not predicate negation. And the Negation Hypothesis (HNEG) seems not valid throughout.

In addition it seems that (11) is ambiguous. It seems possible that weak negation is strengthened to the more informative meaning. This process is called negative strengthening and has gotten a great deal of attention in the literature (see Larry Horn's work, especially Horn (2017)). The main question is how this process can be derived. Horn favors an explanation in terms of pragmatic strengthening. If we have the choice of using a more informative expression over a less informative one we should use it if we can. That a speaker uses a less informative expression in order to mean something more informative is captured as an effect of politeness Horn (2017).

The different meanings may be visualized by means of measurement scales.

On a measurement scale that represents levels of education we may find the King of France (if he existed) either within a zone of indifference (characterized by negation "neither educated nor uneducated", Sapir (1944), c) or below (if he is uneducated, e and d) or above (if he is educated, b and a). (11) (weak negation) would be true the king is e, d, or c. (12) excludes the middle.



2.2 Double negation

There seems to be additional, independent evidence for the difference between weak and strong negation: Double negation does not cancel out sometimes. Horn (1989: p. xiii) states in the introduction to his seminal work that "the absolute symmetry definable between affirmative and negative propositions in logic is not reflected by a comparable symmetry in language structure and language use." Here is Horn's puzzle. Traditionally, the Square of Opposition visualizes relations between propositions expressed by the sentences containing quantified sentences interrelated by contradiction, contrariety and entailment. The propositions may form a diagram (see Parsons 2021, for the history, criticism, modifications and its application to reasoning). Horn uses the diagram in order to group expressions in quadruplets and explains gaps in lexicalization, deriving additional meaning, scalar implicatures (Horn 1989).

The so-called Singular Square of Opposition combines propositions that are expressed by sentences with a individual term in subject position and a scalar predicate, its antonym and their negations (Horn 2017). In the case of predications as in Figure 1 *a* refers to any individual and the different types of predicates are related by negations³: The predicates $\langle educated_A, not$ uneducated_I, uneducated_E, not educated_O serve as a good example for a quadruplet that illustrates the diagram. The different predicates have the common denominator P that may be replaced by the adjective educated. And not-educated is lexicalized as uneducated. But what is the semantics of not-? If not- would have the same function as classical logic, the two negations would cancel out. With respect to our example this would mean that not uneducated and educated mean the same thing. But they do not.

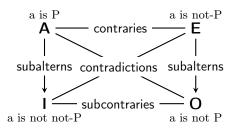


Figure 1: Singular predications

The examples containing the corresponding pairs of antonyms and their negations are related by the same sense relations as in the Traditional Square of Opposition: The A-O corner statements and the E-I corner statements are related by the sense relation contradiction, the A-E corners by contrariety and the I-O corners by sub-contrariety. Furthermore the A-I corners and the E-O corners are related by implication and sometimes called subalterns. The main point is that there is this difference in meaning between **not educated** and **uneducated** that may be expressed by two different types of negation. If its semantics were classical negation, the square would collapse if two classical negation cancel each other out. That is the difference is as-

³The template for the square of opposition in LATEX is from the answer on a question on Stackexchange https://tex.stackexchange.com/questions/594633/square-of-oppositions-diagram.

sumed to be a difference in the semantics of the negation used in lexicalized negative predicates, not classical logical negation. We called this type of negation strong negation (Horn 1989: p. 133). But there are more names for it: non-binary negation, *contrary negation* Concept Negation (following Kant), sometimes Copula Negation (in the Philosophical tradition), Special Negation (Jespersen 1917), and even Constituent Negation (Klima 1964).

Horn refrains from giving a semantics for strong negation. Horn & Wansing (2020) mention (and reject) a proposal for a *quasi-modal notion for contrary negation* akin to logical impossibility. So, the question how strong negation may be captured semantically and how the adjectival cases relate to the quantificational cases is an open question in Horn's work. Bierwisch (1989) argued that antonyms are related not by classical negation, i.e. not truth-value reversal, but *scale reversal*, see also Moeschler (2020). And Löbner (1990: Ch.8., p.165) has an account where antonyms are duals and not contraries of each other which is counterintuitive, at least at first sight. They give rise to contraries or contradictions according to Horn and Lyons.

Note that double negation does not cancel out in the Traditional Square of Opposition. The quantificational determiners $\langle \mathbf{every}_A, \mathbf{some}_I, \mathbf{no}_E, \mathbf{not} \mathbf{every}_O \rangle$ form a quadruplet, for example, and may relate two predicates Sand P. The resulting forms of sentences can be arranged in the corners of the diagram as in Figure 2, left. Usually **some** \mathbf{P} is represented by an existential quantifier. But it could also be represented by its dual as in Figure 2, right. Double negation does not cancel out because the universal quantifier intervenes.

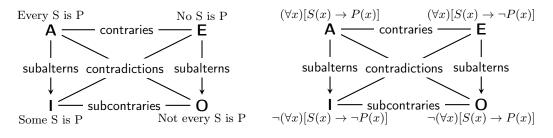


Figure 2: Quantified Sentences

Since the universal quantifier and the existential quantifier are dual operators, the type of negation expressing a contrariety amounts to narrow scope of classical negation with respect to the quantifier and the type of negation expressing a contradiction amounts to wide scope negation with respect to the quantifier used. In the case of quantificational contraries, the difference between contrariety and contradiction is a scope effect of negation and the respective quantifier according to the Laws of Negation. Horn's puzzle that double negation does not cancel out may be related to the problem that the Traditional Square of Opposition and the Singular Square of Opposition don't quite fit together. The next phenomenon is related to the last problem.

2.3 Adjectival Horn Scales

Gotzner, Solt & Benz (2018) argue that the notion of adjectival Horn Scales and Horn's assumption on the Singular Square of Oppositions are not compatible. They claim that "it might be tempting to take Aristotle's square of opposition as a template to be applied to all kinds of Horn scales. However, it is particularly important in the context of adjectival scales that the meaning relations of the square of opposition do not generalize." This is Gotzner et al's puzzle. We may illustrate the point with the adjectival Horn scale $\langle educated, erudite \rangle$, where the sentence a is erudite entails that a is educated. These two adjectives then may be instances of a quadruplet that gives rise to a Singular Square of Opposition. Erudite and educated may be placed in the A and I corner, and their negations in the corresponding E and O corner. But **not educated** is contrary to **erudite**, i.e. more in the sense of **ignorant**, and still it should be contradictory to **educated** simultaneously. But this seems impossible and intuitively wrong. Educated and **not educated** (in the sense of **ignorant**) are contraries. It is unclear what is going wrong here. Gotzner et al. dismiss the value of the Singular Square of Opposition in order to investigate sense relations. I claim that this is not necessary. The problem rather tells us something about how negation interacts with degree morphology.

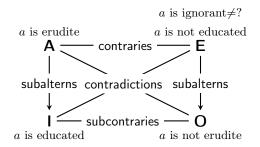
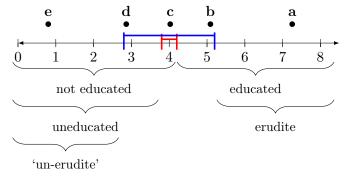


Figure 3: Horn scales squared up with negation

If we map the Square to a measurement scale the problem becomes obvious. erudite is a more extreme adjective than educated and covers the higher end of the scale. This also means that the gap between erudite and its antonym (blue) is widened compared to educated and its negation (red). The extension of not educated is a contrary expression to erudite, so is uneducated and both may be strengthened to the 'real' antonym of erudite, which I introduced as 'un-erudite' in (13). But the gaps considered are variable. Using the more extreme adjective (or stronger adjective) in the vicinity of the more stereotypical (weaker) adjective may turn its negation into a contrary. This process again looks like negative strengthening but of a different type, triggered by the extreme adjective. The number of contextually available gaps seems to enhance strengthenability.



In the following, I explore a theory of gradability more closely that allows for adapting the Singular Square of Opposition to the more traditional version with nominal quantifiers. The main point is that scalar adjectives in the positive are accompanied by a degree quantifier. So strong and weak negation may turn out to be a scope effect. Double negation does not cancel out because the degree quantifier intervenes between the two and for the mapping of extensions of scalar adjectives on a measurement scale to the Square of opposition it is mandatory to keep the domain of quantification constant (but maybe not the force of the quantifier). The last step solves Gotzner's problem with the Singular Square of Opposition.

3 'Strong' negation is truth functional

3.1 Semantics of scalar adjectives

In the following, I introduce a degree account of adjectives with positive morphology. Scalar adjectives associate with a *scale* that is established by an empirical comparison relation Krantz et al. (1971). We may compare individuals with respect to their height, comparing vertical distances from floor to head, or with respect to happiness or dirtiness or preference, whatever. The empirical order on a set of individuals, i.e., quantifiable attributes like height, happiness or temperature and the like, is sometimes called a dimension (Bierwisch 1989, Kennedy 2007).

A central role in natural language semantics is played by *measure functions* in order to capture several aspects of language where numbers seem to play a role. The basic idea is that measure functions are needed in order to interpret scalar adjectives. Instead of saying **Floyd is tall**, we could have said **Floyd is six feet tall**. Adding a number and a unit of measurement to a scalar adjective may make a statement less 'vague' Morzycki (2020). A scalar adjective should mean the same thing, however, whether it is paired with a measure phrase like **six feet** or not. In the accounts of the Cresswell-Heim-vonStechow school, scalar adjectives are relations between individuals and degrees (on a contextually restricted scale). The truth conditions of an adjective introduce a suitable measure function that assigns the individual a degree on a scale and this degree is compared to other degrees. The dimension of the scale that an adjective rests on specifies the measure function μ_{DIM} .

The lexical entry for **educated** as relevant in Jacobs' example above (see Section 2.1) introduces a constant **EDU** together with a situation variable i, as in (13). It stands in for a relation between individuals and degrees as in (14).

- (13) $|\mathbf{educated}| = \mathbf{EDU}_i$
- (14) $\llbracket \mathbf{EDU}_i \rrbracket^g = \lambda d.\lambda x.\mu_{\text{EDUCATION}}(g(i))(x) \ge d,$ where g(i) is the utterance situation.

The meaning of the adjective collects the values that the individual gets from the measure function and all degrees below.⁴ The set of degrees characterized by the adjective for each individual is therefore a considerable initial part of a scale. That is, scalar adjectives are downward-monotonic.⁵ Measure phrases are captured as names for degrees or quantifiers Schwarzschild (2002).

The positive form of an adjective is composed of an operator **POS** (if there is no measure phrase) and the meaning of the adjective, where **POS** is a quantifying degree argument of the adjective. I adopt the view from von Stechow (2009a) that the degree quantifier relates two sets by subset relation, i.e. it is a universal quantifier.⁶ The first set is the one characterized by the degree predicate. The second set is a contextually determined interval NORM, somewhere in the middle of the scale, the zone of indifference. Von Stechow calls it the *delineation interval*. And the second set is supposed to be contained in the first. This operator corresponds to a universal quantifier as defined in (15). The positive operator is of Type (dt)t, i.e. a quantifier over degrees.

(15) $|\text{POS}| = \lambda D^{(dt)} \cdot (\forall d) [\text{NORM}_i(d) \rightarrow D(d)]$

⁴The version of adjective semantics as relations between a degree and an individual is from Heim (2001) who elaborates on Cresswell (1976). Kennedy (1999) defended the view that adjectives denote just measure functions (and not relations). In this view, an adjective assigns to an individual a certain value — the measure — from the set of reals. This view goes back to Bartsch & Vennemann (1972) and is evaluated in Heim (2001). DeClerq & Wyngared (2017) add a monotonicity operator Q to the basic meaning of adjectives that leads to a set of values that are below the measure. Their account is also Kennedian.

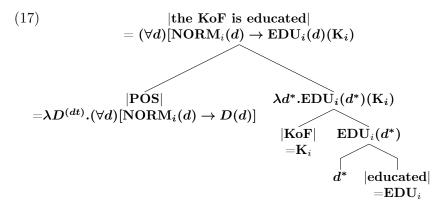
⁵A similar assumption can be found in Seuren (1978), as DeClerq & Wyngared (2017) notice. Seuren uses a paraphrase like "the e (extent) that is attributed to an individual" in order to refer to a set of degrees.

⁶Again this way of looking at the interpretation of the positive may be traced back to Seuren (1978).

Norm is a function that singles out the zone of indifference on the scale relevant in the situation of evaluation. Its intension is defined as in (16).

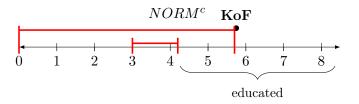
(16) $[[NORM]]^g = \lambda s^s \cdot \lambda d^d \cdot d$ is in the (relevant) zone of indifference in s.

Being a degree quantifier, the argument **POS** cannot be combined semantically with the adjective meaning directly by functional application. One way to solve this problem is that **POS** undergoes movement at the level of Logical Form.



The King of France is educated is true if the interval that represents the NORM in a context c, the zone of indifference, is covered by the degrees that precede (or are equal to) the measure of the king. The truth conditions are exemplified in (18). The underbraced area is usually called the extension of the adjective in question. The king of France's measure is in that area.⁷

(18) Educated



We will look at how \mathbf{POS}_i interacts with negation in its various forms in the next section.

3.2 Strong and weak negation

Since the extensions of scalar adjectives are relations that assign a truth value to a pair of individuals and degrees, our earlier definition for affixal

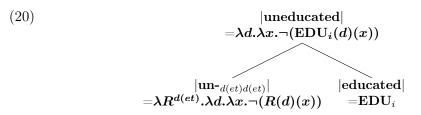
⁷We assume that the King of France exists in s^* and in c

negation as stated in (8) cannot be used in order to combine directly with the extension of a scalar adjective. (8) can only be applied to one-place non-gradable adjectives. Applying it to a gradable predicate would result in a type-mismatch. But a variant of classical logical negation will do (see also von Stechow 2009b)): We change the type of negation to a modifier of scalar adjectives as in (19). It is evident that negation in this definition for degree negation obeys the Negation Hypothesis stated in (1) above. The basis for the application of this negation is again classical logical negation. This is what Heim (2008) originally introduced as adjectival negation. Typeflexibility of negation extends to adjectival negation.⁸

$$|\mathrm{un-}_{d(et)d(et)}| = \lambda R^{d(et)}.\lambda d.\lambda x.
eg (R(d)(x))$$

Negation has the effect of reversing the perspective on which degrees are characterized by the degree predicate but the ordering of the scale remains untouched.⁹ The meaning of the adjective does not collect the measure that the individual gets from the measure function *but all degrees above*. The set of degrees characterized by the adjective for each individual cover, therefore, a considerable end part of a scale.

Below the word-level, affixal negation may modify the degree predicate directly, as illustrated in (20) (internal negation).



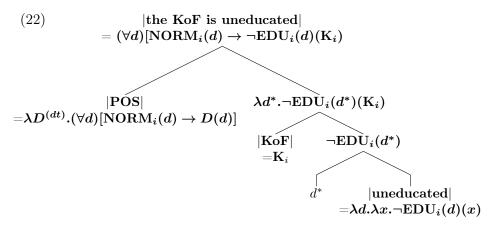
The negation effects the greater-than-equal relation. The positive adjective and its negation relate to the same dimension or scale. There is no scale reversal.

⁸And the same may be said for negative polar adjectives that are implicitly negative. They are decomposed into the adjectival root and negation, (see also DeClerq & Wyngared 2017: for adjectival negation).

⁹Seuren (1978) intended such an analysis. But he still needs some kind of scale reversal in order to talk about negative extents. He uses a paraphrase like "the e (extent) that is not attributed to an individual." But this description is not uniquely defined: Nothing prevents us from looking at extents that start at the beginning of the scale and end somewhere in the middle. There are many such extents. But we are interested in the complement. DeClerq & Wyngared (2017) elaborate on that proposal, using the strategy of the Heim-von Stechow interpretation for negation. In their account, negation just adds set complementation. And their way of interpreting positive-polar and negative-polar scalar adjectives in the Positive is equivalent to the one laid out here. In the Comparative, however, the interaction with nominal and modal quantifiers in object position is problematic, as has been already argued for with respect to Kennedy's approach by Heim (2001). Kant (1977[1763]: 783f.) speaks about 'negative Größe' and means negative extents.

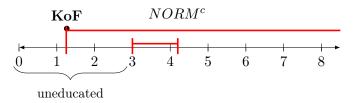
(21)
$$[\lambda d.\lambda x.\neg (\mathbf{EDU}_i(d)(x))]^g = \lambda d.\lambda x.\mu_{\mathrm{EDU}}(s)(x) < d$$

Above the word level, the positive operator, i.e. a degree quantifier, is generated in the position of the degree argument of the adjective and raised to the left periphery of the sentence for interpretation, leaving a trace whose value is bound on the path of the interpretation of quantifier raising. This is illustrated in (22). But affixal negation remains bound to the word level.



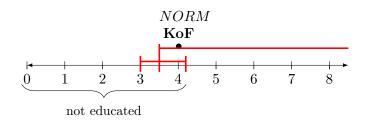
The King of France is uneducated is true if the interval that represents the NORM in a context c, the zone of indifference, is covered by the degrees that follow (or are equal to) the measure of the king. The truth conditions are illustrated in (23). Negative extents start somewhere on the scale and end in infinity.

(23) Uneducated

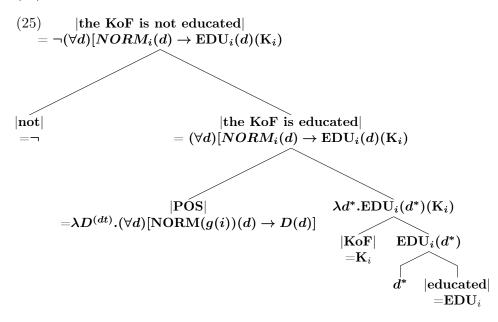


Sentential negation, however, may take scope above the positive operator. Negating a universal quantifier may result in an existential statement. There are degrees among the zone of indifference such that the King of France does not have them. These truth conditions are compatible with the illustration in (23) and in (24). Sentential negation is less informative.

(24) Not educated



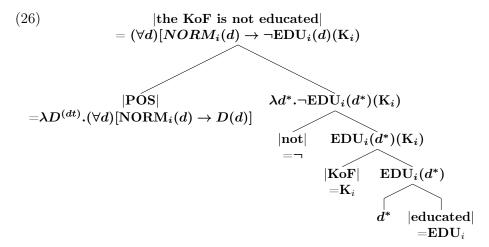
This strategy of interpretation represents the weak reading as illustrated in (25).



The difference in meaning between affixal negation in terms of degree negation as defined above and sentential negation turns out to be a difference in scope of NEG, the meaning of classical negation, with respect to the **POS**-Operator. If the **POS**-Operator has wide scope with respect to negation, a weak reading is derived. If the **POS**-Operator has narrow scope with respect to negation, a strong reading is derived.

3.3 Negative Strengthening: a scope effect?

The **POS**-operator may also outscope negation, as illustrated in (26), rendering the construction with negation ambiguous. The interpretation of the **POS**-operator having wide scope with respect to negation turns out to be equivalent to the strong reading of affixal negation: contradictory **not educated** may be strengthened to contrary **uneducated** giving rise to what is called *negative strengthening*. In the account here, negative strengthening is a scope effect and in principle optional. Negation may be sentential (type tt) as in (26) or it modifies lower in the tree, i.e. the verbal phrase (type (et)(et)) with no difference in meaning because the subject is not quantificational.

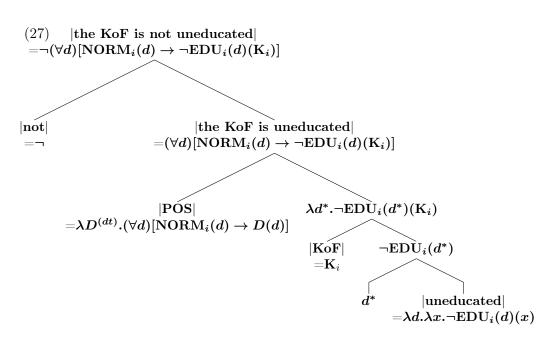


But we will see below that this view on negative strengthening as scope inversion is to simple minded. Pragmatics may play a role, in addition, in the case of double negation.

3.4 The Semantics of Double Negation

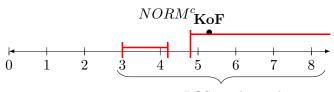
Israel (2011: p. 88) discusses litotic expressions and distinguishes between two different readings of litotes: *attenuation* and *understatement*. So there are different uses of the rhetoric figure litotes, and it seems these correspond to the two scope relationships between sentential (or predicate) negation and the degree operator **POS** under the assumption that **POS** is mobile at LF. Scopal issues alone cannot explain the pattern of attenuation or understatement, however. In this section, only the semantics of double negation is of interest and the kind of semantic strengthening we observed with single negation. But, the account is compatible with Neuhaus (2019) finding, that doubly negated scalar adjectives are four times ambiguous. I am going to argue that one type of ambiguity is grounded in scope inversion (reading 1 and reading 2) of the **POS**-operator and sentential negation. Both resulting readings may be pragmatically enriched (to a reading 1^{*} by attenuation and a reading 2^{*} leading to understatement).

Reading 1 Double negation does not cancel out if sentential negation is interpreted with wide scope with respect to the positive operator and affixal negation has to be interpreted with narrow scope because it is generated word internally. This reading is represented in (27). (27) is true if the King of France's measure is among the values of **NORM**_i or higher.



This reading illustrated in (28). If the King of France has an education that counts as not uneducated the sentence (27) can be true if the King is in fact educated.

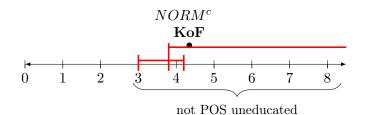
(28) Not POS uneducated (Reading 1a)



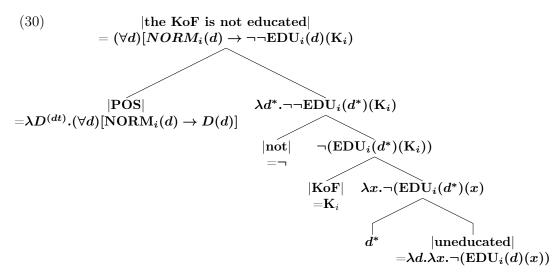
not POS uneducated

This reading is the basis for what is called attenuation. Krifka (2007) and many others argue that the extension of a doubly negated scalar adjective may cover a medium or mild state of the measured property. But (27) does not exclude that the measure of the King of France could lay within the **NORM** interval as illustrated in (29) and the pragmatic question remains why it should lay there if double negation is used. Some kind of strengthening seems to take place and I will come back to pragmatic effects below.

(29) Not POS uneducated Reading (1b)

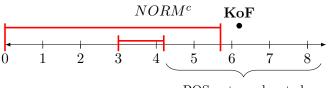


Reading 2 Constructions with double negation are ambiguous, as well. If the **POS**-operator takes wide scope with respect to sentential negation, sentential negation and affixal negation end up being adjacent and since both are truth-functional, they cancel out. This predicts that **not uneducated** and **educated** may have the same truth conditions.



These truth conditions are compatible with the illustration in (31). Again scope inversion strengthens the meaning of double negation (a little bit) by excluding degrees within the zone of indifference. And it changes the direction of looking at the measure intended.

(31) **POS** not uneducated



POS not uneducated

Again this is obviously not the whole story. The actual effect described in the literature is that the expression with double negation actually leads to an *understatement*, which amounts to considerable strengthening. It is often not just equivalent with the positive form. What a speaker means by using **not uneducated** in this case is that the level of education could be *extraordinally high*.

What we observe, so far, is that scope inversion may lead to some kind of strengthening, i.e. semantic strengthening in the case of positive-polar and negative-polar adjectives. But it seems that pragmatics plays a role on top. That is there are different types of negative strengthening: *semantic negative strengthening* (=scope inversion), the right branch in the illustration in Figure 4. In addition, both readings may undergo further processes of interpretation that we may call *pragmatic negative strengthening*.

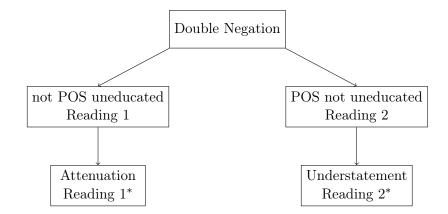


Figure 4: Double Negation gets four readings

In the next steps, I approach the process of pragmatic strengthening and the problem of Adjectival Horn Scales that seem not to match with measurement scales.

4 Sense relations across different types of domains

4.1 Singular and Traditional Square of Opposition are family

The method of interpretation featured in this paper relates in an interesting way to the visualizations of sense relations in the Square of Oppositions. The operator **POS** is a universal quantifier quantifying over degrees. It may occur without negation, with narrow scope negation (internal), with wide scope negation (external) or both as illustrated in Figure 5, for any gradable predicate P and individual a. The restriction of the universal degree quantifier, **NORM**_i, is held constant if part of the Square. In the following, I ignore the additional readings derived by **POS**-movement. Wide scope readings would lead the Square to collapse to one contradiction.

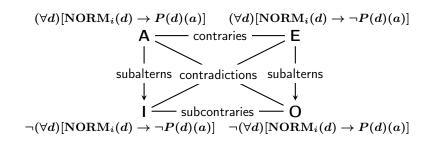


Figure 5: Nominal quantification

For educated instantiating P and related to any individual a, we may get a quadruplet in the sense of Horn: $\langle educated_A, not uneducated_I, une$ $ducated_E, not educated_O \rangle$. These four instances related by internal and external negation and the **POS**-operator may be called a word field. **Educated** and **not uneducated** are related by entailment in the usual sense of Horn and represent a (positive) Adjectival Horn Scale. **Uneducated** and **not educated** are related as well and represent a (negative) Adjectival Horn Scale. Strong and weak negation correspond to internal and external negation.

If scalar adjectives are in fact quantificational because of a silent **POS**operator, the two types of Squares of Opposition (Traditional and Singular) are instances of the same type of Square. Where the traditional version for nominal quantification is usually symbolized with universal and existential quantification over individuals, as in Figure 6. The sense relations hold if the domain restriction of the quantifiers is not empty.

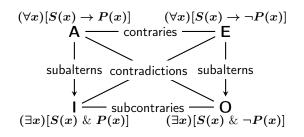


Figure 6: Nominal quantification

Formulas containing existential quantifiers \exists are interdefinable by formulas containing \forall . The interdefinability of the quantifiers (and other constants) with the help of external and internal negation is extensively discussed in Zwarts (1991). One instance of the Laws of Negation is (32a) that relates the I corners of the Traditional and the Singular Square of Opposition and another one is (32b) that relates the O-corner.

(32) a.
$$(\exists x)[Sx \& Px] \equiv \neg(\forall x)[S(x) \to \neg P(x)]$$

b.
$$(\exists x)[S(x) \& \neg P(x)] \equiv \neg (\forall x)[S(x) \to P(x)]$$

In English, every S and some S are duals of each other in the nominal domain. No S is called the contradual of every S. Double negation of a universal quantifier corresponds logically to the dual of the universal, i.e. the existential quantifier — eliminating all negations (= theory of quaternality, see also (Gottschalk 1953, Löbner 1990)). According to the Laws of Negation we observe equivalences like (33), for example. The equivalences become clearest in sentences describing episodes. Think of a relevant set of cats in a situation of choice. If (33a) is true, (33b) is true as well and vice versa, under all circumstances.

- (33) a. No cat slept (yesterday).
 - b. Every cat didn't sleep (yesterday).

It is important to note with Zwarts (1991) that in the nominal domain there is more than one way to express negation if a sentence is quantified. This is a consequence of duality. There is no 1-1 correspondence between positive and negative sentences as soon as quantification is involved.

4.2 Two word fields for *possible*

So far, I only considered nominal and degree quantification. Modal operators are quantificational, as well, and these expressions can be related by means of the Square of Opposition, as well. **Possible** seems ambiguous between at least a scalar reading and a non-scalar one.

Non-scalar reading: If **possible** is not gradable it may serve as the existential counterpart to **necessary**, i.e. it's dual, and it may have negative forms like **not possible** and **impossible**, respectively, meaning the same thing. In this reading, the quadruplet is $\langle \text{necessary}_A, \text{possible}_I, \text{not possible}_E, \text{not necessary}_O \rangle$ and the expressions modify a proposition, as in Figure 7. Negation cancels out. Possible and not impossible mean the same thing. In this reading, **possible** and **not possible** give rise to a contradiction.

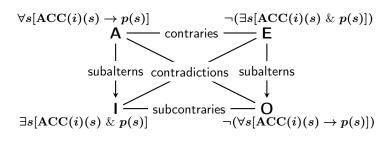


Figure 7: Modal Operators

The lexical entry for **possible** as relevant in the non-scalar, modal reading introduces the expression a quantificationally complex representation, as in (34). Its interpretation is usually captured as an existential quantifier over possible situation (or worlds) where ACC(g(i)) characterizes the set of accessible situations from the situation of evaluation g(i) as defined in (35).

- (34) $|\text{possible}_{modal}| = \lambda p^{(st)} \exists s[\text{ACC}(i)(s) \& p(s)], type((st)t)$
- (35) $[[\mathbf{ACC}_i]]^g = \lambda s.s$ is an accessible alternative to g(i), where g(i) is the situation of evaluation.

The lexical entry for **necessary** is defined accordingly as a dual of **possible**, i.e. a universal modal quantifier. And negation is sentential negation interpreted classically.

Scalar reading: If possible is scalar, it turns out to be modified by an implicit **POS**-operator. If so, the quadruplet is $\langle possible_A, not impossible_I, impossible_E, not possible_O \rangle$. In this reading, possible and impossible give rise to a contrariety, as in Figure 8. And double negation does not cancel out because the **POS**-operator intervenes. But negation itself is either internal or external negation and is interpreted classically, as well.

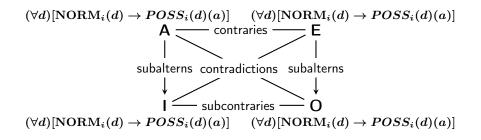


Figure 8: Two Possibles

The degree variant is represented in (36). There, a measure function is part of its meaning that measures probabilities, as in (37). Its interpretation not quantificational. It relates propositions with respect to how probably it is that they are true in the situation of evaluation g(i). The quantificational force in this case is due to the (silent) positive operator.

- (36) $|\mathbf{possible}_{deq}| = \mathbf{POSS}_i, \text{ type } (d((st)t))$
- (37) $[[\mathbf{POSS}_i]]^g = \lambda d.\lambda p.\mu_{\text{POSS}}(s)(p) \ge d$

In this case, **not impossible** turns out the be a dual of **possible** (if combined with **POS**, i.e. corresponding to existential quantification by the laws of negation (as well). It is obvious that language acquisition is challenging, as far as **possible** and its scale-mates is concerned. It is a difference in types. And silent quantifiers may be at work that are only acquired in contrast to comparative and superlative.

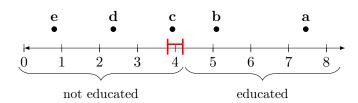
Only in the degree reading is **possible** combinable with **well**. This intensifier pushes the probability of the proposition p towards 100 percent if the degrees measured by **possible**_{deg} are positive and towards 0 percent if the degrees are negative. This treatment also matches Kennedy & McNally (2005) observation that **well** is associated with closed scales. **well** seems to manipulate the standard of comparison or in other words: it widens the gap.

5 Variation in the 'Zone of Indifference'

5.1 Adding Extremeness

In the next step, I would like to consider how the Square of Opposition relates to measurement scales used with scalar adjectives in many accounts. **Educated** and **not educated** may partition the scale associated with measurements of levels of education in roughly two parts: the one above and the one within and below the zone of indifference. Consider the illustration in (38). The upper bound is the (maximal) cut-off point. But interlocutors may differ in how and where exactly they imagine the zone of indifference on the scale.

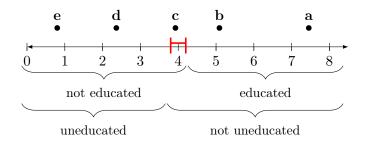
(38) Educated and not educated



We may place the zone of indifference that **educated** is evaluated against, i.e., the values of **NORM**_i, somewhere in the middle of the scale (red). This interval may comprise borderline cases of being educated, for instance, and could be rather small or bigger. It at least consists of one cut-off point if minimally small, but we do not know which point is the cut-off point. In von Stechow's account, borderline cases fall under the (wide scope) negation of **POS**-operator of the degree predicate and therefore count as false and not undefined. The cut-off point between the extensions of the gradable attributes is the upper bound of the interval. Individuals that have a degree of education greater than the zone of indifference (here a and b) count as educated, all the others don't.

Adding internal negation, it becomes obvious that the cut-off point changes to the lower bound of the interval. Degrees lower as this point count as **uneducated** and the complement as **not uneducated**. 'Being not uneducated' turns out to be less informative than 'being educated'. This is what we derive from the semantics so far. It is the normal, stereotypical interpretation of negated expressions as illustrated in (39).

(39) Uneducated and not uneducated



The illustration changes if we add an intensifier like **very** to **educated**. We may get the quadruplet: $\langle \text{very educated}_A, \text{ not very uneducated}_I, \text{ very uneducated}_I, \text{ very uneducated}_E, \text{ not very educated}_O \rangle$.

Very is defined as in (40), following in spirit again von Stechow (2009a). The idea is that using **very** changes the zone of indifference from a smaller interval to a bigger one that contains the small one symmetrically but otherwise very is similar to the **POS**-operator. In this sense, **very** is a domain widener.¹⁰ It may translate as in (40).

(40)
$$|\text{VERY}_i| = \lambda D^{(dt)} . \exists I[\text{WIDEN}_i(\text{NORM})(I) \& (\forall d)[I(i)(d) \rightarrow D(d)]]$$

Norm was defined in (16), above, as a function that singled out the zone of indifference on the scale relevant in the situation of evaluation. Its intension was (41).

(41) $[NORM]^g = \lambda s^s \cdot \lambda d^d \cdot d$ is in the (relevant) zone of indifference in s.

WIDEN may operate on **NORM**, as in (42).

(42) $\llbracket WIDEN \rrbracket^g = \lambda s.\lambda T^{s(dt)}.\lambda I^{s(dt)}.I$ is relevant in s and the set of degrees characterized by I(s) symmetrically includes the set of degrees characterized by T(s) and the first is considerably bigger than the second.]

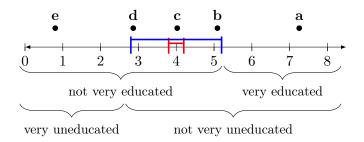
In the situation of evaluation, it takes a **NORM**-function that singles out the zone of indifference and makes it considerably and symmetrically bigger.¹¹

¹⁰I would like to see **very** in analogy to **any**, in some sense, in the nominal domain. Kadmon & Landman (1993: p. 361) argued that **any** is a domain widener. And widening has to do with quantities in their account, as well.

¹¹In von Stechow's definition the existence of the widened interval is part of a presupposition. I leave clarifying this point for further research. The existence of the widened

Applying very to educated widens first of all the gap at issue between the very educated and its antonym. Speaker and hearer may agree on any interval that includes the NORM in *i*, i.e. the widened *I* (blue) as in (43). Not very educated corresponds to the contradiction and very uneducated corresponds to the contrary. And very uneducated finds a contradictory expression in not very uneducated.

(43) very educated and very uneducated



The corresponding Square of Opposition is illustrated in Figure 9. An individual that is not very educated like b, c, d may also count as not very uneducated. An individual that is not very uneducated like b, c, d may also count as not very educated. The opposite inference is called Scale reversal in Gotzner et al. account. In my account these inferences are entailments.

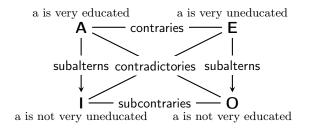


Figure 9: very and the Square of Oppositions

5.2 The Dual of the Positive Operator

This discussion about **very educated** allows for a fresh look on the question how Horn Scales relate to antonymy. Recall that Gotzner et al. present

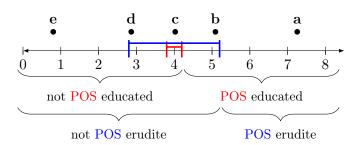
(i) A: The king of France is very educated. B: # No that's not true. He is educated. B*: Hey wait a minute. He is educated.

interval is not at-issue. It is non-deniable and it projects under negation (see Ebert 2024: for discussion of test for non-at-issueness in connection with appositives and gestured information.). Consider (i), for example, an instance of non-deniability.

work on adjectives that are part of adjectival Horn Scales as $\langle educated$, erudite \rangle , for example. They, in fact, reject that the quadruplet $\langle erudite_A$, educated_I, not educated/ignorant_E, not erudite_O \rangle are instances of word fields giving rise to a Square of Opposition.

Let us start with the assumption that **erudite** and **very educated** mean more or less the same thing. And consider the illustration in (44). If **erudite** and **very educated** mean the same thing, then **erudite** is evaluated against a wider zone of indifference (blue) than **educated** (red) in the last subsections. But whoever is erudite is also educated. So it seems that **erudite** and **educated** are indeed scale-mates in the sense of Horn, where **erudite** is stronger or more informative and **educated** is weaker and less informative. And as scale-mates they could form quadruplets of expressions related by contradiction and contrariety that may visualized by means of the Square of Opposition. Scale-mates are related by entailment. **Educated** may even turn out to be a lexicalization of **not 'unerudite'** (which is ungrammatical).

(44) Educated and erudite



With respect to this picture there are degrees among the degrees belonging to the wider gap (blue) such that individuals who are educated to those degrees definitely count as educated: those degrees that are higher than the upper bound of **NORM**_i (red) and lower than the upper bound of widened **NORM**_i (blue). This fact about (44) may be stated in (45) for the individual b. The **POS**-operator of **educated** may be re-interpreted in combination with the alternatives building on **erudite/very educated** as an *existential quantifier* that quantifies over the wider gap —- triggered by the alternative **erudite**.

(45) |b is educated|
=
$$(\exists d)[\text{NORM}_i(d) \& \text{EDU}_i(d)(b)]$$

The presence of the stronger alternative **erudite** is responsible for the evaluation of **educated** with respect to the wider gap. Widening the gap weakens the force of the **POS**-Operator for the weaker alternative in an adjectival Horn Scale.

The assumption that there is an existential version of the degree quanti-

fier associated with positive morphology in addition to a universal quantifier for degrees (as argued by Heim and von Stechow), fits the theory of quaternality (see Gottschalk and other logicians). I would like to claim that the use of **educated** in the vicinity of **erudite** is independent evidence for the conclusion in natural language that expressions may be force variable. Scalar adjectives in the positive may turn out to be force variable like other quantificational expressions in the modal domain (Rullmann, Matthewson & Davis 2208, Deal 2011). Rullmann and colleagues and Deal are concerned with modals in languages that do participate in Horn Scales. It seems that scalar adjectives have the choice. If they are weak, a universal quantifier may become an existential one by widening the restriction of the quantifier.

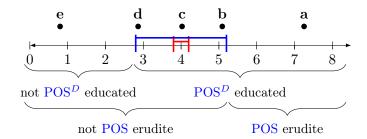
I will call the implicit existential form 'POS-DUAL-ITIVE' based on the idea that its translation is the dual of the **POS**-operator: POS^{D} . This operator is defined as in (46) on the basis of its universal (dual) counterpart. The choice of $NORM^{c}$ is dependent on the choice of the zone of indifference of the alternatives that are relevant in c.

(46) Definition Pos-dual-itive $|\mathbf{POS}^{D}| = \lambda D.(\exists d) [\mathbf{NORM}_{i}(d) \& D(d)]$

5.3 Negating the existential: a form of negative strengthening

The alternative adjectives associate with different sections on the scale of measurement. Compare (44) and (47). Not POS educated with respect to the original zone of indifference (red) has an existential counterpart not \mathbf{POS}^{D} educated, but with respect to the widened zone of indifference. This existential counterpart accounts for the strengthened reading, equivalent to ignorant. Erudite and not erudite use the universal degree operator with respect to the widened gap.

(47) Educated, erudite and widening



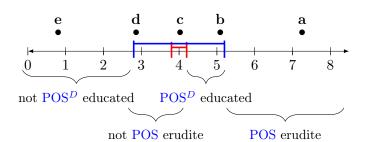
5.4 Towards Attenuation

The existential reading is really weak. It is predicted that even degrees that do not count as educated — where only **not educated** is the alternative are degrees that could count as educated: Those degrees below the upper bound of smaller zone of indifference (red). This is problematic. But note that widening the zone of indifference has the consequence that for a rather considerable part of the widened gap *we are unsure* whether we should count somebody as educated or not - out of the blue, i.e. whether we choose our original setting, the stereotypical one, where only **educated** and **not educated** count as alternative expressions or whether we look at the paradigm of alternatives in the quadruplet related to **erudite**. The variable choice of possible cut-off points from different possibilities of partitioning the same scale in fact *increases* the vagueness of existential **educated**.

Therefore, it seems pragmatically reasonable to only consider degrees above the original (smaller) gap as degrees that count definitely as being educated. This idea is consistent with the assumptions from epistemic theories of vagueness (Williamson 1994) and can be captured as a special case of Gricean Reasoning (Krifka 2007), namely as a tendency towards stereotypical interpretations. "The urge to use expressions that both interlocutors are likely to use in the same way for the classification of phenomena is somewhat reminiscent of the tendency towards stereotypical interpretations that underlies implicatures based on what has been called the R principle by Horn (1984, 1993) and the I principle by Atlas & Levinson (1981) and Levinson (2000). The R/I principle is generally seen as leading to an enrichment of the literal meaning of expressions such that they refer to stereotypical instances." Eliminating vagueness caused by widening may explain why the weaker alternative **educated** definitely covers only the upper part of widened gap.

This reasoning might be paired with general considerations about the strength of expressions. If somebody uses a weaker term (existential educated, for example) and he could have used a stronger one (universal erudite) then by the Q principle we might conclude that the speaker uses the weaker term because she knows that the stronger term is not applicable. This explains why educated only covers the upper middle section of the scale. The Q principle also reduces the area of application of not erudite to the area of the widened gap. But in addition we may use the M principle: not erudite is more complex than existential educate. So educate wins over not erudite for the upper part of widened gap. The resulting picture is illustrated in (48). This reasoning is found in Krifka (2007).

(48) Pragmatic effects caused by widening



We may conclude that widening the gap, existential quantification for the weaker term plus pragmatic reasoning explains the partition of the scale into four parts: the segment that is covered by **erudite** (positive extreme), the segment that is covered by **educated**, the segment that is covered by **not erudite** and the segment that is covered by **not educated**. In addition, **ignorant** may be used in order to name the negative extreme on the scale, replacing existential **not educated** in the strengthened meaning. In that case **POS ignorant** is a lexicalization of **POS not-erudite**.

But, **ignorant** may turn out to be not equivalent to **not educated**. It only depends on the alternatives that are relevant in the discussion of the situation of utterance but also on whether Gricean reasoning takes place or not and of course the choice of the zone of indifference is variable, anyway.

Observe, moreover, that there is a subtile difference in meaning between not educated interpreted existentially with respect to the widened zone of indifference and uneducated interpreted universally with respect to the smaller (original) zone of indifference. Not educated as the negation of existential educated is stronger than uneducated. Not educated is soto-speak strenghtened by widening the gap and this process is well known for negative polarity items. There are some widening approaches to negative polarity items (especially Kadmon & Landman (1993), see Giannakidou (2011) for intensive discussion). The process of widening the gap, triggered by erudite being a relevant alternative may turn parts of the sequence of educated into a NPI (in the vicinity of negation) and a free choice element elsewhere. Educated maybe related to erudite like some is to every, or in the combination with negation to any.

The consequence of this proposal is that there are two instances of educated, one that combines with an existential quantifier as in (48) and one that does not as in (38) on p. 23 above. Only the existential one is part of the Horn Scale. All arguments that we know form pragmatic reasoning concerning scalar imlicatures carry over easily. Negating the existential \mathbf{POS}_i^D automatically strengthens **not educated** to **ignorant**. Importantly the negation in **not educated** has wide scope with respect to the Pos-dualitive. It could even be that the degree quantifier in its existential guise is merged with the sentential negation preceding it.¹² Consider the translation

 $[\]overline{}^{12}$ Thanks to Elena Herburger for this comment. This fact needs more investigation and

in (49).

(49) $|\mathbf{not} \ \mathbf{POS}^D| = \lambda D^{d(et)} \cdot \lambda d^e \cdot \neg (\exists d) [\mathbf{NORM}_i(d) \& D(d)]$

5.5 Adjectival Horn Scales and the Singular Square of opposition

Erudite and **not educated/ignorant** are contraries. **Educated** and **not educated** are contradictions. Gotzner and colleagues' problem is a problem of lexical ambiguity so to speak, whether the weaker term **educated** in a Horn Scale is combined with a existential or a universal quantifier or, in other words, whether it is interpreted in the sense of stereotypical **educated** or as a lexicalization of **not un-erudite**. But it is obvious that we have to chose the existential one in the Square of Opposition if part of a Horn Scale since the entailment patterns discussed by Horn are based on the assumption that the restrictions of the quantificational expressions do NOT vary. The Square of Opposition then can be stated as in 10.

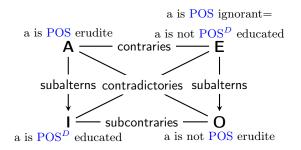


Figure 10: The Horn Scales on **erudite** squared up

The system of sense relations seems further evidence for the sometimes existential nature of the Positive Operator and, in addition, we also conclude that the learner has double evidence for a possible existential interpretation of an otherwise universally quantified element. Similarly, it is argued for the double nature of free choice items and negative polarity items, in van der Wouden (1997), for example Penka (2021).

Which adjectives are force variable and which are not, I leave open for future investigation. But it seems that the weak ones in the sense of Gotzner et al. those that pair up with more extreme adjectives are good candidates. Furthermore, the observed effect that the pragmatic function of negated adjectives is difficult to pin down for informants (Gotzner & Kiziltan 2022) is explained by the variability of the zone of indifference dependent on what the the alternatives considered are. It could have to do with how wide the zone of indifference is chosen. In addition, this work relates to work on

historical considerations may shed light on the development.

extreme adjectives (Morzycki 2012): Those that allow for widening combine with **very**, for example, and occur in the comparative, those that don't do not combine with **very**.

And after all, contra Gotzner, Solt & Benz (2018), it has been shown that measurement scales and the idea of Horn Scales, are each compatible with the Square of Opposition.

5.6 Attenuation and understatement

So far, we looked at adjectival Horn Scales with adjectives with different adjectival roots. But doubly negated elements seem to pair up with their unnegated scale-mates, as well. **Erudite** and **educated** stand in the same relation as **educated** and **not uneducated**. Whereas in the first pair universal **erudite** meets existential **educated**, in the second pair universal **educated** meets doubly negated universal **educated**.

Consider the four sentences in (50).

- (50) a. The king of France is educated.
 - b. The king of France is not educated.
 - c. The king of France is uneducated.
 - d. The king of France is not uneducated.

The propositions that these four sentences express may be related to the Square of Opposition, as in Figure 11.

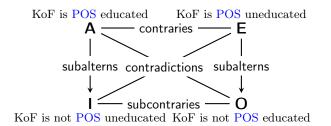
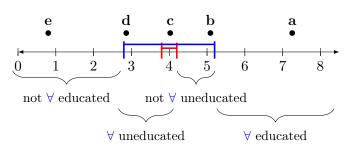


Figure 11: The Horn Scales on educated squared up

In order to account for the attenuating effect (Reading 1^{*}), we can assume that the interpretation is tied to a more widened gap if two negations are involved. I would not go as far as to claim that widening of the gap is iconic to double negation. But it seems that the use of the internal negation **un**- in addition to sentential negation may have the widening effect (see also Krifka 2007: for similar reasoning). For the four parts on the scale associated with **educated** and **not uneducated** we may just argue in the same way as for the pair **erudite** and **educated**, above. Attenuation is a combination of the semantics of double negation in addition with a process of widening the zone of indifference in order to eliminate vagueness phenomena for the simpler expressions. This widening triggers additional pragmatic reasoning. Attenuation of **not uneducated** is illustrated in (51). **not uneducated** may cover the same area on the educatedness scale as existential **educated** in the last section. Attenuation of double negation follows in this system. Pragmatic strengthening has to do with how Q, RI and M principle interact.

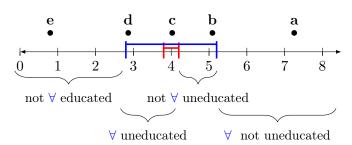




What is still open is how double negation may end up being interpreted as an extreme positive polar attribute, i.e. how understatement (Reading 2^{*}) could be explained. If widening of the zone of indifference took place (in order to eliminate the danger of vague expressions, see Krifka), **educated** becomes a more extreme extension. And then, scope inversion may lead to some kind of strengthening that I called semantic strengthening, above. If gap widening has taken place, **educated** gets a more extreme meaning than when it is just contrasted with sentential negation. If this is so, then it seems that scope inversion also explains understatement (modulo widening), namely, if the doubly negated form is interpreted with respect to the widened gap (blue) and compared with stereotypical **educated** evaluated with respect to the smaller gap (red). Its extension assimilates to the one of **erudite**.

Understatement and negative strengthening are two consequences of the same process, namely scope inversion between the degree operator and sentential negation. That this is so, is blurred by the effect of the variability of the gap.

(52) Understatement



Taken together, attenuated **not uneducated** and understatement **not undeducated** cover the same area on the education scale as **educated**. The difference between the two expressions is a difference in the zone of indifference.

6 Polarity Asymmetries

The account so far predicts that a construction with a sentential negation in combination with a scalar adjective is ambiguous between a weak and a strong reading and that double negation may cancel out but it doesn't have to, namely if the positive operator intervenes between the two negations. In other words, we observe the following strengthening patterns in (53) (single negation pattern) and (54) (double negation pattern) as an effect of scope inversion.

- (53) NEG pos-pol Adj. \Rightarrow neg-pol Adj. negative strengthening
- (54) NEG neg-pol Adj. \Rightarrow pos-pol Adj. strengthening to the positive (=understatement)

I distinguished semantic strengthening and pragmatic strengthening. Semantic strengthening, i.e. scope inversion already covers those two patterns. This difference is not made in the literature, however. In this section, I would like to look at explanations for negative strengthening in the literature and compare them with my approach.

Horn (1989) already discusses different types of strengthening. With respect to double negation he is concerned with (a) attenuation and argues that this may be conceived as a violation of the Maxime of Quantity (the speaker has not enough evidence to claim the plain unnegated form). The doubly negated form might be strengthened to cover the *complete* zone of indifference that is characteristic for antonyms. The stronger form seems to be negated. Note that this type of effect has nothing to do with our two patterns in (54) and (53).

The explanation turns out to be an instance of regular pragmatic strengthening of the meaning as derived in Section 3.4 where the **POS**-Operator has intermediate scope with respect to the two negations, a scalar implicature. The stronger alternative is implicated to be false. Krifka (2007), however, argues that this is not quite correct. He gives the following examples. (55a) may mean (attenuating) that the speaker was rather happy (55b) and not neither happy nor unhappy.

- (55) a. I was not unhappy to find that my kids were already awake. (Krifka 2007: ex.3ab)
 - b. I was quite happy to find that my kids were already awake.

Note that (55a) may also be understood as an understatement, meaning that the speaker was very happy. This reading may be enforced by adding an NPI like **at all**. The strengthening effect of at all is also mentioned in Horn (2017).

(56) I was not unhappy, at all, to find that my kids were already awake.

In my account the effect of attenuation is an interplay of R/I principle and M principle and widening of the zone of indifference basically following Krifka (2007). And it is correctly predicted that a doubly negated element may be used to mitigate the unnegated form if widening is assumed and vagueness eliminated.

(b) But Horn considers (55) and (56). According to Horn (1989: p. 358), see also Brown & Levinson (1987), negative strengthening could be socially motivated. A common explanation for single negation patterns is that a weaker expression like **not educated** is used as a face-saving practice even if the stronger **uneducated** would be more informative. We use a weaker expression in order not threaten the addressee. In my account this type of strengthening is semantic and an instance of scope inversion of the scope of the degree operator and sentential negation. The literature reports *polarity* asymmetries: That a negated positive-polar adjective (55) is "strengthened" is more likely than that a negated negative-polar adjective is "strengthened" (56). This generalization is very robust (Ruytenbeek, Verheyen & Spector 2017). not uneducated is (usually) not easily strengthened to highly educated. And it remains to be tested whether the intonation may help to get the reading with scope inversion. In my account, this finding could be related to cognitive complexity issues. The understatement reading uses widening of the zone of indifference which should be contextually based, it uses scope inversion (like the negative strengthening cases of simple negation cases) and it uses negation cancellation of two occurrences of negation. The simple negation cases only use scope inversion.

In more recent time, Mazzarella & Gotzner (2021), Gotzner & Mazzarella (2021), and Gotzner & Kiziltan (2022) investigated the face threatening aspect of negative strengthening experimentally more deeply and found that it is mainly the adjectival polarity (internal negation) that is responsible for the strengthening pattern. Importantly, they show that social reasoning seems to have a lesser effect. They identify scale structure in the sense of Kennedy & McNally (2005) as a factor in order to derive implicatures (absolute adjectives are not vague, for example) and the complexity of the morphology — morphologically complex adjectives like **not unhappy** are strengthened less likely compared to **not sad**. And there are gender differences.

Furthermore they looked at adjectival Horn Scales and found that, all in all, negated expressions seem to have less pragmatic functions compared to bare ones — it seems that relational adjectives are more inclined to cover the middle portion of a scale as soon as sentential negation is involved. Bare adjectives however are associated with specific areas on the scale. **Happy** may cover a higher segment than **content** where the two negated forms cover the same segment closer to the middle section. It seems that their informants only get the attenuating reading, the one where the **POS**-Operator intervenes between the two negations. This reading is cognitively less costly.¹³ The understatement reading involves raising of the **POS**-Operator and widening of the quantifier domain in addition. The attenuating reading only involves pragmatic reasoning on the side of the hearer.

Gotzner and Mazzarella also speculate that this may have historical reasons. In fact, affixal **un**- underwent a change from a pure negative marker form the verbal domain to a negative marker in the adjectival domain and this change may be coincident with a system change, where the **POS**-Operator becomes a meaning part of the negative affix which renders the **POS**-Operator immobile in the vicinity of double negation. I will quickly come back to this issue below on page 36. We may conclude nevertheless that the readings with wide scope POS operator with respect to two negations are available. Some people get them (they are accompanied with domain widening) and they are attested in the literature (Israel 2011: and many others). But how exactly the lexicon affects the availability of these readings is an open question.

There seems an interesting difference in interpretation dependent on the adjectival root, combining a negative-polar adjective with double negation.¹⁴ In (57) the inner negation seems just to be dropped for interpretation, an effect reminicent of Negative Concord constructions. One interesting point on these examples is that sentential negation seems to license the **un**-affixation. **nicht untraurig** means 'rather happy', some kind of attenuation. Note that the 'rather happy' reading of **not unhappy** could be captured by existential quantification and widening of the zone of indifference in the system proposed here. And it might well be that force variability of the **POS**-operator is at the root of this phenomenon.

- (57) a. Papa, ich bin nicht untraurig darüber, dass wir morgen heimfahren", erklärte sie mir an unserem letzten Abend im Hotel.¹⁵ 'Papa, I am not sad [lit. not un-sad] about that we go home tomorrow, she said to me on our last evening at the hotel.'
 - b. Freuen wir uns: Das Beste für Familie Raffael! Wir sind nicht untraurig über dein Fehlen. Zumal es ein toller Grund ist!¹⁶

¹³It has also been argued that negation has a mitigating effect (Zuanazzi et al. 2024: most recently).

¹⁴German examples are provided by Helmut Weiß, p.c.

¹⁵Wechselbad der Gefühle - und Abenteuerurlaub - Erlebnis.net; https://www.erlebnis. net > wechselbad-der-gefuehle

 $^{^{16}}$ Borussia on Twitter: "Herzlichen Glückwunsch, Raffael, zur ...; https://twitter.com \rightarrow borussia \rightarrow status

'Let's be happy: the best for the Raffael family! We are not sad [lit. **not un-sad**] about your absence. Especially since it's a great reason.'

Horn (1989: p. 280) mentions comparable examples (e.g., Germ. **unzweifel**los 'without doubt' or Engl. **irregardless**) which 'contain redundant or pleonastic (rather than mutually annihilating) double negation'. In addition, the phenomenon of 'too many negations' is also known from the literature on so called depth-charge sentences like **No head injury is too trivial to be ignored**. And the fact that these sentences get the readings they get and are not compositional may have to do with processing issues.

Sometimes double negation may still cancel out even if the adjective is actually scalar and ungrammatical if sentential negation is dropped. These are cases of understatement and maybe just grammaticalized.

- (58) a. das in seiner heute noch aktuell wirkenden Aussage nicht unheikel ist [...]. Oper Frankfurt.¹⁷
 'that is not unproblematic in its statement which still seems to be relevant today.'
 - b. Allerdings: "Ein Austausch ist nicht unheikel", meint Alessandro Panella, [...]. So berge er die Gefahr^{'18}
 'However: An exchange is not unproblematic, says Alessandro Panella, [...]. So it carries the risk.'

Furthermore, it is predicted that double negation always eliminates if the predicate negated doubly is not gradable like **married** above. Although, sometimes even non-gradable adjectives may be doubly negated and their meaning shifts. Consider (59). In (59), **married** could be coerced into a scalar adjective, that is true if one is more or less close to marriage, generating a measure function that allows for comparison of individuals and the degrees they have on that scale. In this case double negation seems to trigger the gradable interpretation.

(59) I am not unmarried.

If the negative affix is able to coerce an intersective adjective into a gradable meaning, it can be defined as in (60). Consider the well known difference in meaning between **not American** and **un-American**. (60) requires a scalar version of the adjective **American**.

(60)
$$|\text{POS-un}| = \lambda D^{d(et)} \cdot \lambda x^e \cdot (\forall d) [\text{NORM}_i \to \lambda d \cdot \neg (D(d)(x))]$$

¹⁷Oper - Der Bauer als Held | deutschlandfunkkultur.de; https://www. deutschlandfunkkultur.de > oper-der-bauer-al.

¹⁸eues Brandt-Kind: Rotwangiger Wonneproppen gehört zur ...; https://www. handelsblatt.com > handel-konsumgueter > n...

But if the **POS**-Operator is processed as a lexical part of the negative morpheme, it might be less mobile which could also explain the tendency of negated negative-polar adjectives not to strengthen their meaning to the corresponding negative one.

7 Conclusion

The theory of the interpretation of scalar adjectives uses a degree semantics that originates with Heim (2008) and von Stechow (2009a) and has its predecessors in Seuren (1978). Negation is type-flexible. Adjectival negation (modifying adjectives) and sentential/predicate negation only differ in the semantic type but not in function. Negation is a truth functional operator. An invisible degree operator **POS** is claimed to be the argument of scalar adjectives. Movement of the **POS** operator may lead to additional readings (scope inversion). There are some cases where it seems that the invisible **POS**-Operator is re-bracketed with affixal **un-** and coerces adjectival roots into gradable attributes.

The new contribution in this paper is that the degree operator may be force variable. Whereas von Stechow assumed that the degree operator is a universal quantifier (see also DeClerq & Wyngared 2017), I show that an existential version (triggered by widening the zone of indifference) is used if adjectival Horn Scales are considered.

I started out with three puzzles: (a) That there are different types of meaning for negation (strong and weak) was reanalyzed as a scope effect: an interaction of the degree operator and negation. If the invisible degree operator has wide scope with respect to negation a strong reading of negation emerges. The Negation Hypothesis put forward by Jabobs remains valid. Even strong negation may be captured as an instance of classical logical negation. There is no need for a contrary negation or maybe one can reanalyze contrary negation as a lexicalization of adjectival negation and the degree operator. That is, negative expressions like **un-** may turn out to be semantically more complex. (b) That double negation often does not cancel out has its reason in the fact that the degree operator usually intervenes between the two negations. If this is so negation cancellation is unexpected. (c) The sense relations where account for in assuming that the degree operator is force variable. Force variability is triggered by the choice of the zone of indifference, in particular widening of the zone of indifference. The POS-Operator may be an existential or a universal degree quantifier.

Strengthening patterns follow from scope inversion of negation and the degree operator (only available if sentential negation is involved) and in addition by pragmatic reasoning. As far as pragmatic reasoning is concerned, I followed basically the explanations from Krifka (2007). His main point is that the zone of indifference that basically everybody assumes in the inter-

pretation of antonyms might be variable and for strengthening patterns, we consider different choices of zones of indifference. Widening of the gap has a strengthening of gradable attribute as a consequence. This also accounts for attenuation and understatement readings in cases of double negation (Neuhaus 2019: see also). The apparent illogical behavior of scalar adjectives and negation may be an effect of the choice of the gap.

One and the same adjective may participate in different entailment scales, though. This was shown with the ambiguity of **possible** and also in a different respect with the ambiguity of **educated** that turned out to be interpreted with variable force. The Square of Opposition does generalize to adjectival meanings. There is no need for illogical negation.

An open question remains what the presupposition cancellation potential is of adjectival negation. In almost all cases negation has narrow scope with respect to all other material in the sentence (except the adjective) and this makes it difficult to cancel a presupposition triggered scope external material. A notable exception is the adjective **inexistent**.

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